Rotational Energy Problem Solving

2002M2. The cart shown above is made of a block of mass m and four solid rubber tires each of mass m/4 and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia \( \frac{1}{2} ML^2 \), where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the total rotational inertia of all four tires.
b. Determine the speed of the cart when it reaches the bottom of the incline.
c. After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance \( x_m \) the spring is compressed before the cart and bumper come to rest.
d. Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of \( x_m \) in part (c). Give a reasonable explanation for this decrease.

1986M2. An inclined plane makes an angle of \( \theta \) with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height h above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is \( \frac{2}{5} MR^2 \). Express your answers in terms of M, R, h, g, and \( \theta \).

a. Determine the following for the sphere when it is at the bottom of the plane:
   i. Its translational kinetic energy
   ii. Its rotational kinetic energy
b. Determine the following for the sphere when it is on the plane.
   i. Its linear acceleration
   ii. The magnitude of the frictional force acting on it

The solid sphere is replaced by a hollow sphere of identical radius R and mass M. The hollow sphere, which is released from the same location as the solid sphere, rolls down the incline without slipping.

c. What is the total kinetic energy of the hollow sphere at the bottom of the plane?
d. State whether the rotational kinetic energy of the hollow sphere is greater than, less than, or equal to that of the solid sphere at the bottom of the plane. Justify your answer.
As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

- **Disk:** mass = 3m, radius = R, moment of inertia about center \( I_D = 1.5mR^2 \)
- **Rod:** mass = m, length = 2R, moment of inertia about one end \( I_R = \frac{4}{3}(mR^2) \)
- **Block:** mass = 2m

The system is held in equilibrium with the rod at an angle \( \theta_0 \) to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of m, R, \( \theta_0 \), and g.

a. Determine the tension in the string.

The string is now cut, and the disk-rod-block system is free to rotate.

b. Determine the following for the instant immediately after the string is cut.

i. The magnitude of the angular acceleration of the disk

ii. The magnitude of the linear acceleration of the mass at the end of the rod

As the disk rotates, the rod passes the horizontal position shown above.

c. Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.
1994M2. A large sphere rolls without slipping across a horizontal surface. The sphere has a constant translational speed of 10 meters per second, a mass $m$ of 25 kilograms, and a radius $r$ of 0.2 meter. The moment of inertia of the sphere about its center of mass is $I = \frac{2mr^2}{5}$. The sphere approaches a $25^\circ$ incline of height 3 meters as shown above and rolls up the incline without slipping.

a. Calculate the total kinetic energy of the sphere as it rolls along the horizontal surface.

b. i. Calculate the magnitude of the sphere's velocity just as it leaves the top of the incline.
   ii. Specify the direction of the sphere's velocity just as it leaves the top of the incline.

c. Neglecting air resistance, calculate the horizontal distance from the point where the sphere leaves the incline to the point where the sphere strikes the level surface.

d. Suppose, instead, that the sphere were to roll toward the incline as stated above, but the incline were frictionless. State whether the speed of the sphere just as it leaves the top of the incline would be less than, equal to, or greater than the speed calculated in (b). Explain briefly.

1991M2. Two masses, $m_1$ and $m_2$ are connected by light cables to the perimeters of two cylinders of radii $r_1$ and $r_2$, respectively, as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is $I = 45 \text{ kg}\cdot\text{m}^2$. Also $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.

a. Determine $m_2$ such that the system will remain in equilibrium.

b. Determine the angular acceleration of the cylinders.

c. Determine the tension in the cable supporting $m_1$.

d. Determine the linear speed of $m_1$ at the time it has descended 1.0 meter.
Two identical spheres, each of mass $M$ and negligible radius, are fastened to opposite ends of a rod of negligible mass and length $2l$. This system is initially at rest with the rod horizontal, as shown above, and is free to rotate about a frictionless, horizontal axis through the center of the rod and perpendicular to the plane of the page. A bug, of mass $3M$, lands gently on the sphere on the left. Assume that the size of the bug is small compared to the length of the rod. Express your answers to all parts of the question in terms of $M$, $l$, and physical constants.

a. Determine the torque about the axis immediately after the bug lands on the sphere.

b. Determine the angular acceleration of the rod-spheres-bug system immediately after the bug lands.

c. The angular speed of the bug

d. The angular momentum of the system

e. The magnitude and direction of the force that must be exerted on the bug by the sphere to keep the bug from being thrown off the sphere
A solid cylinder with mass $M$, radius $R$, and rotational inertia $\frac{1}{2}MR^2$ rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height $H$. The inclined plane makes an angle $\theta$ with the horizontal. Express all solutions in terms of $M$, $R$, $H$, $\theta$, and $g$.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane. Your arrow should begin at the point of application of each force.

c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is $\left(\frac{2}{3}\right)g\sin\theta$.

d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

e. The coefficient of friction $\mu$ is now made less than the value determined in part (d), so that the cylinder both rotates and slips.

i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part (a). Justify your answer.

ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.

Answers:

- **2002M2** a. $I_w = 4I = \frac{1}{2}mr^2$; b. $v = \sqrt{\frac{8}{5}}gh$; c. $x_m = 2 \sqrt{mg/hk}$.
- **1986M2** a. i. $\left(\frac{5}{7}\right)Mgh$ ii. $\left(\frac{2}{7}\right)Mgh$; b. i. $(5/7)g\sin\theta$; ii. $(2/7)g\sin\theta$.
- **1999M3** a. $T = 5mgsin\theta$; b. i. $\alpha = (6g\sin\theta)/(13R)$ ii. $a = (12g\sin\theta)/(13)$; c. $v = 4\sqrt{(3gR\cos\theta)/(13)}$.
- **1994M2** a. 1750 J; b. 7.56 m/s, 25° above horizontal; c. 7.93 m.
- **1991M2** a. $m_2 = 20/3$ kg; b. 2.0 rad/s$^2$; c. 180 N.
- **1992M2** a. 3Mgl; b. 3g/5l; c. $\omega = \sqrt{(6g/l)}$; d. $\sqrt{30M^2gl}$; e. 6.6 Mg upward.
- **1997M3** a. $\sqrt{(4/3)gH}$; c. $(2/3)g\sin\theta$; (1/3)tan$\theta$; e. translational speed greater, total kinetic energy less.