Chapter 18: Electromagnetic Induction

- How might electrical stimulation of the brain help treat certain diseases?
- How does a microphone work?
- How does a vending machine sort coins?

Make sure you know how to:
1. Find the direction of the \( \vec{B} \) field produced by an electric current (Section 17.7).
2. Find the direction of the magnetic force exerted on moving electric charges (Section 17.5).
3. Explain how an electric field produces a current in a wire and how that current relates to the resistance of the wire (Sections 16.2 and 16.4).

Metal wires are not the only type of material capable of carrying electric current. Body tissue, including brain cells, is also electrically conductive. Inside the human brain are cells called neurons, which contain ions that can move in much the same way as free electrons move in wires. Brain activity is electrical in nature, and for some time, scientists have believed this property of the brain could be used to help treat certain diseases. However, the skull is a fairly good electrical insulator, and until recently, the only options for electrical stimulation of the brain were to either apply a very high potential difference across points on the skull or to surgically implant electrodes into the brain. A promising new way to study and alter the electric activity in the brain without using dangerously high potential differences or invasive electrodes is called transcranial magnetic stimulation (TMS). TMS technology may help treat diseases such as clinical depression, Parkinson’s disease, and Huntington’s disease. The amazing part of this technology is that it is noninvasive. A physician or scientist simply holds a small coil of wire on or near the patient’s scalp. A power source produces an abrupt electric current through the coil, which in turn produces a localized current in the brain directly under the coil, even though there is no electrical connection between the outside coil and the brain. How is this possible?

[Lead] In the last chapter, we learned that an electric current through a wire, or moving electrically charged objects in general, produces a magnetic field. Could the reverse happen? Could a magnetic field produce a current? It took scientists many years to answer this question. In this chapter, we will investigate the conditions under which this can happen.

18.1 Inducing an Electric Current

In transcranial magnetic stimulation (TMS), an abrupt change in electric current through a coil outside a patient’s skull produces an electric current inside the brain. Since there is no direct electrical contact between the coil and the brain, what causes the current in the brain? In Chapter 16, we learned that an electric current results when a battery or some other device produces an
electric field in a wire. The field in turn exerts an electric force on the free electrons in the wire connected to the battery. As a result, the electrons move in a coordinated manner around the circuit—an electric current.

In this section we will learn how to produce a current in a circuit without a battery—called *inducing* a current. We start by analyzing some simple experiments in Observational Experiment Table 18.1. The experiments involve a bar magnet and a coil. The coil is connected to a galvanometer that registers the current. The coil is not connected to a battery. See if you can find any patterns in the results of the experiments.

**Observational experiment Table 18.1 Inducing an electric current using a magnet**

<table>
<thead>
<tr>
<th>Observational Experiments</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>You hold the magnet motionless in front of a coil.</td>
<td>The galvanometer reads zero.</td>
</tr>
<tr>
<td>You move the magnet toward the coil.</td>
<td>In either case, the galvanometer needle moves to the right, indicating a current through the coil.</td>
</tr>
<tr>
<td>Alternatively, you move the coil toward the magnet.</td>
<td></td>
</tr>
<tr>
<td>You move the magnet away from the coil.</td>
<td>In either case, the galvanometer needle moves to the left, indicating a current through the coil but in the opposite direction than in the last experiment.</td>
</tr>
<tr>
<td>Alternatively, you move the coil away from the magnet.</td>
<td></td>
</tr>
<tr>
<td>You turn the magnet 90° in front of the coil.</td>
<td>In both cases, the galvanometer registers the same of current.</td>
</tr>
<tr>
<td>Alternatively, you turn the coil 90° in front of the stationary magnet.</td>
<td></td>
</tr>
<tr>
<td>You collapse the sides of the coil together so its opening becomes very small.</td>
<td>In both cases, the galvanometer registers a current, but the direction is different in each case.</td>
</tr>
<tr>
<td>You pull open the sides of the collapsed coil so the area becomes large again.</td>
<td></td>
</tr>
</tbody>
</table>

**Patterns**

Although no battery was used, an electric current was induced in the coil when either the magnet or the coil moved toward or away from the other. Current was also induced when the coil’s orientation relative to the magnet or the shape of the coil changed.

In Chapter 16 we found that there is a current in a closed circuit if the circuit is connected to a battery. The battery is an external source of emf that produces an electric field in the circuit.
In the experiments in Observational Experiment Table 18.1 there was no battery yet the galvanometer registered an electric current through a coil. What produced the emf in the experiments in Observational Experiment Table 18.1?

Recall from Chapter 17, that a magnetic field can exert a force on moving electrically charged particles. The force exists only if the magnetic field or a component of the field is perpendicular to the velocity of the electrically charged particles. Let’s consider again the experiment in Observational Experiment Table 18.1 in which the coil moves toward the magnet; for simplicity, we use a square loop made of conducting wire (Fig. 18.1). Inside the wire there are positively charged ions that make up the crystal lattice of the metal and negatively charged electrons. The ions aren’t free to move but some of the electrons are. The $\vec{B}$-field produced by the bar magnet points to the left and spreads out as shown in Fig. 18.1.

![Figure 18.1](image)

**Figure 18.1** Motion of metal loop toward bar magnet causes magnetic force on electrons in loop.

Notice in the figure that at the top and bottom sections of the loop, a component of the $\vec{B}$-field is perpendicular to the velocity toward the right of those sections of wire. As a result, the magnetic field exerts a force on each electron in the wire. These forces cause the electrons in the wires to accelerate clockwise around the loop (use the right hand rule for the magnetic force for the negatively charged electrons.) Note also that electrons in the front vertical section of the loop accelerate upwards, while the electrons in the back vertical section accelerate downwards. The overall result is that due to the relative motion between the loop and the bar magnet, the electrons start moving around the loop in a coordinated fashion—we have an induced electric current.

Thus, it seems like the currents produced in the experiments in Observational Experiment Table 18.1 can be explained using the understanding of the magnetic force that we developed in Ch. 17. Can we use this understanding to explain how transcranial magnetic stimulation (TMS) works? We can’t. In the TMS procedure, the coil is not moving relative to the brain, whereas our previous explanation requires motion of the loop relative to the magnetic field.

Perhaps there is another explanation. Let’s examine the experiments in Observational Experiment Table 18.1 from a different perspective, one that focuses more on the $\vec{B}$-field itself rather than any sort of motion. When the magnet or coil moved or rotated with respect to each
other, the number of \( \vec{B} \)-field lines going through the area of the coil increased or decreased (Fig. 18.2a). The number of \( \vec{B} \)-field lines through the coil also changed when the area of the coil changed (as in Fig. 18.2b where the area of the coil is collapsed.) Thus an alternative explanation for the pattern we observed in the experiments is that when the number of \( \vec{B} \)-field lines through the coil’s area changes, there is a corresponding emf produced in the coil, which leads to the induced current.

![Figure 18.2](image)

Figure 18.2 Changing \( B \) and \( A \) causes induced current (a) as bar magnet gets closer, more \( B \) field lines thru loop (b) as area of loop decreases, fewer lines thru loop

We can summarize these two explanations for the induced current as follows:

**Explanation 1:** The induced current is caused by the magnetic force exerted on moving electrically charged objects (for example, the free electrons in conducting wires that are moving relative to the \( \vec{B} \)-field).

**Explanation 2** Any process that changes the number of \( \vec{B} \)-field lines through a coil’s area induces a current in the coil. The mechanism explaining how it happens is unclear at this point.

Let’s test these explanations using new experiments. The first explanation suggests that motion is the important condition. The second explanation suggests that changing the \( \vec{B} \)-field through the coil is the important condition. Therefore, an experiment involving a change in the number of \( \vec{B} \)-field lines through a coil’s area, but with no relative motion, can be used to test these explanations. As you remember, there are multiple ways to create a magnetic field. We could use a permanent magnet or a wire that carries current (Chapter 17). The coil in Fig. 18.3a has a \( \vec{B} \)-field that resembles the \( \vec{B} \)-field of the bar magnet in Fig. 18.3b.

![Figure 18.3](image)

Figure 18.3 Magnetic fields due to current loop and bar magnet are the same
The testing experiments in Testing Experiment Table 18.2 use two coils. (Fig. 18.4a) Coil 1 on the bottom is connected to a battery and has a switch to turn the current through coil 1 on and off. Current in coil 1 produces a magnetic field whose $\vec{B}$-field lines pass through coil 2’s area (Fig. 18.4b). For each of the experiments we can use the two explanations to predict whether or not there should be an induced electric current in coil 2.

![Figure 18.4](image)

**Figure 18.4** Experiment to decide if magnetic field from coil 1 induces current in coil 2

<table>
<thead>
<tr>
<th>Testing experiment</th>
<th>Prediction</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>The switch in the circuit for coil 1 is open. There is no current through coil 1.</td>
<td>Based on Explanation 1 due to motion: There is no current through coil 1, thus there is no magnetic field at coil 2. Neither coil is moving. No current will be induced in coil 2.</td>
<td>The galvanometer registers no current through coil 2.</td>
</tr>
<tr>
<td>You close the switch in the circuit for coil 1. We are interested in the time interval during which the switch is being closed. The current through coil 1 increases abruptly, from zero to a steady final value.</td>
<td>Based on Explanation 2 due to changing field through coil: There is no current through coil 1 while the switch is open; therefore there is no change in the number of $\vec{B}$-field lines through coil 2’s area, and there should be no induced current through coil 2.</td>
<td>Right when the switch closes the galvanometer needle briefly moves to the left then returns to vertical, indicating a brief induced current through coil 2.</td>
</tr>
<tr>
<td>You keep the switch in the circuit for coil 1 closed. The current through coil 1 has a steady value.</td>
<td>Neither coil is moving, thus no current will be induced through coil 2.</td>
<td>The galvanometer registers no current through coil 2.</td>
</tr>
</tbody>
</table>
You open the switch again. We are interested in the time interval during which the switch is being opened. The current through coil 1 decreases abruptly, from its steady value to zero.

Neither coil is moving, thus no current will be induced through coil 2.

The decreasing current through coil 1 produces a decreasing $B$-field. This changes the number of $B$-field lines through coil 2’s area. There should be a brief induced current through coil 2.

Right when the switch opens the galvanometer needle briefly moves to the right (opposite the direction in experiment 2), then returns to vertical, indicating a brief induced current through coil 2.

**Conclusion**

The predictions based on Explanation 2 matched the outcomes in all four experiments. The predictions based on Explanation 1 were incorrect for two of the four experiments. Motion is not necessary to have an induced current. In contrast, a change in the number of $B$-field lines through a coil’s area is always accompanied by an induced current through that coil. Explanation 1 has been disproved, but Explanation 2 has not.

We learned that it is possible to have a current in a closed loop of wire without using a battery. This phenomenon of inducing a current using a changing $B$-field is called **electromagnetic induction**.

Let’s apply this understanding to the transcranial magnetic stimulation (TMS) discussed in the opening passage of the chapter. When the physician closes the switch in the circuit containing the coil outside the patient’s skull, the increasing current in the coil produces a changing $B$-field within the brain, which in turn induces current through the brain’s electrically conductive tissue. A current is briefly and non-invasively induced in the brain.

**Conceptual Exercise 18.1 Moving loops in steady magnetic field** Figure 18.5 shows four loops moving at constant velocity $\vec{v}$ relative to a region with a constant $B$-field (within the dashed line). In which of these loops will electric currents be induced?

**Figure 18.5** Four rectangular loops moving relative to magnetic field

*Sketch and Translate* An electric current will be induced in a loop whenever the number of $B$-field lines through the loop’s area changes.

*Simplify and Diagram* For each case, current flow should be as follows:
(1) The loop is moving into the field region, so the number of $\vec{B}$-field lines through its area is increasing. A current will be induced through the loop.

(2) A current will be induced through the loop for the same reasons as (1).

(3) The loop is completely within the field region, thus the number of $\vec{B}$-field lines is not changing. As a result, no current is induced.

(4) No current is induced for the same reason none is in (3).

*Try It Yourself:* What happens to loops 3 and 4 as they are leaving the magnetic field region?

*Answer:* The number of $\vec{B}$-field lines through each loop will then be changing and a brief current will be induced as the loops leave the $\vec{B}$-field region.

**Discovery of electromagnetic induction**

We observed electromagnetic induction with relative ease. However, the first observation of this phenomenon in 1831 was difficult. Following Hans Oersted’s discovery in 1820 that an electric current produces a magnetic force on a compass needle, scientists wondered if the reverse might occur—could an object with magnetic properties produce an electric current? In 1821, British experimentalist Michael Faraday set out to do this, and it took him 10 years to succeed. Two difficulties lay before him—one conceptual and one technical. The conceptual difficulty was that although a steady current always produces a magnetic field, a steady magnetic field does not always produce an electric current. The technical difficulty was the need for coils rather than single loops of wire. At the time, physicists did not have the materials or the procedures to produce insulated wires, which are needed to make coils of wire (the individual wires can’t be in contact with each other.) The galvanometers of the time could not detect the weak currents induced in a single loop.

Finally, Faraday and American physicist Joseph Henry discovered electromagnetic induction. Henry devised and published a method for insulating wires by wrapping them in silk. He then used insulated wire to make a coil. During the process, he observed that a current was induced in the coil. Henry did not immediately publish his discovery. Upon learning about Henry’s wire insulation method, Faraday constructed coils. In 1831, he conducted experiments similar to those described earlier in this section and published his findings. Since then, scientists and engineers have devised many practical devices based on electromagnetic induction, such as electric generators, magnetic card readers for credit cards, coin sorters for vending machines, the transformers needed for modern electric power grids, magnetic brakes for automobiles, and the pick-up coils for string and percussion instruments.

**Dynamic microphone and seismometer**

A dynamic microphone that converts sound vibrations into electrical oscillations employs the principle of electromagnetic induction. (Fig. 18.6) When a sound wave, such as a singer’s voice, strikes the diaphragm inside the microphone, the diaphragm oscillates. A coil of wire attached to the diaphragm moves alternately closer and farther from a magnet in the microphone,
corresponding to locations with stronger and weaker magnetic field. This changing magnetic field through the coil induces a changing current in the coil. This changing current representing the sound can then be used to store the details of the sound electronically.

Another application that operates by the same principle is a seismometer, which is a sensor that detects seismic waves during an earthquake. The seismometer has a massive base with a magnet that sits on the Earth and vibrates as seismic waves pass. At the top, a coil attached to a block hangs at the end of a spring (Fig. 18.7). The spring acts as a shock absorber that reduces the vibrations of the hanging block and coil. The motion of the base relative to the coil induces a current in the coil, which produces a signal that is recorded on a seismograph.

Review question 18.1
Your friend thinks that relative motion of a coil and a magnet is absolutely necessary to induce current in the coil that is not connected to a battery. Describe an experiment you could perform to disprove your friend’s idea.

18.2 Magnetic Flux
In the last section we found that an electric current is induced in a coil when the number of \( \vec{B} \)-field lines through the coil’s area changes. This occurred when:

- The strength of the \( \vec{B} \)-field in the vicinity of the coil changed.
- The area \( A \) of the coil changed.
- The orientation of the \( \vec{B} \)-field relative to the coil changed.

The goal of this section is to construct a physical quantity that quantifies the number of \( \vec{B} \)-field lines through a coil’s area. Based on the analysis in the last section, changes in that
quantity should cause an electric current to be induced in the coil. Ultimately, we want to be able to predict the direction and magnitude of the induced current. Physicists call this physical quantity magnetic flux $\Phi$.

Qualitatively, the magnetic flux can be understood as the number of $\vec{B}$-field lines passing through a particular two-dimensional area (which often is the area inside a wire loop, but the loop itself is not needed for the flux to exist). If the magnitude of the $\vec{B}$-field throughout the area is greater, the number of field lines through the area is greater. Additionally, if the area itself is larger, the number of field lines through the area is greater. This suggests that the magnetic flux through the area is proportional to the magnitude of the $\vec{B}$-field throughout the area, and proportional to the size of the area itself. Mathematically:

$$\Phi \propto BA.$$ 

How do we include the effect of the orientation of the loop through which we are calculating the flux relative to the $\vec{B}$-field lines into the above equation? Imagine a rigid loop of wire in a region with uniform $\vec{B}$-field. If the plane of the loop is perpendicular to the $\vec{B}$-field lines, then a maximum number of field lines pass through the area (Fig. 18.8a). If the plane of the loop is parallel to the $\vec{B}$-field, then zero field lines pass through the area (Fig. 18.8b). In between these two extremes, the magnetic flux takes on intermediary values (Fig. 18.8c). In other words the magnetic flux through the area depends on the orientation of the area relative to the $\vec{B}$-field lines.

![Figure 18.8 Flux depends on angle between normal line to loop surface and magnetic field direction](image)

To describe this relative orientation we use a line perpendicular to the plane of the loop—the dashed normal line shown in Fig. 18.8. The angle $\theta$ between this dashed line and the $\vec{B}$-field lines quantifies this orientation. Since the cosine of an angle is maximum when the angle is zero, and minimum when the angle is 90 degrees, it seems like the magnetic flux through the area is also proportional to $\cos \theta$. This leads to a precise definition for the magnetic flux through an area.
Magnetic flux The magnetic flux $\Phi$ through a region of area $A$ is

$$\Phi = BA \cos \theta$$

(18.1)

where $B$ is the magnitude of the uniform magnetic field throughout the area and $\theta$ is the angle between the direction of the $B$-field and the direction of a line drawn perpendicular to the area (the dashed lines in Fig. 18.8). The SI unit of magnetic flux is the unit of the magnetic field (the tesla T) times the unit of area ($m^2$), or $T \cdot m^2$. This unit is also known as the weber (Wb).

Equation (18.1) assumes that the external magnetic field is uniform. For situations in which this is not the case, you first split the area into small sub-areas within which the $B$-field is approximately uniform; then add together the magnetic fluxes through each. This book will not address such cases.

Note that one does not need a loop of wire to calculate a flux through a particular area.

Tip! The word ‘flux’ in everyday life refers to something that is undergoing change. “Our plans for the weekend are in flux.” This is not what is meant by the word as used in physics. The word flux means the value of a particular physical quantity. When the $B$-field, the orientation of the loop, and the area of the loop are constant, the flux through the area of the loop is constant.

In Section 18.1, we proposed that a changing magnetic field induces current through a wire loop. We can now refine that idea and say that current is induced when there is a change in the magnetic flux through the loop’s area. In other words, if the magnetic flux throughout the loop’s area is steady, no current will be induced.

**Quantitative Exercise 18.2 Flux through a book cover** A book is positioned in a uniform $B$-field whose magnitude is 0.20 T and points from left to right in the plane of the page (Fig. 18.9). Each side of the book’s cover measures 0.10 m. Determine the magnetic flux through the cover when: a) the cover is in the plane of the page; b) the cover is perpendicular to the plane of the page, and whose normal line makes a $60^\circ$ angle with the $B$-field; and c) the book’s cover area is perpendicular to the plane of the page, and whose normal line points toward the top of the page (Fig. 18.9.)
Represent Mathematically The magnetic flux through an area is determined by Eq. (18.1), \( \Phi = BA \cos \theta \) The angle in each of the three situations is: (a) \( \theta = 90^\circ \), (b) \( \theta = 60^\circ \), and (c) \( \theta = 90^\circ \).

Solve and Evaluate The magnetic flux through the book cover in each case is:

(a) \( \Phi = (0.20 \, \text{T})(0.10 \, \text{m})^2 \cos(90^\circ) = 0 \).

(b) \( \Phi = (0.20 \, \text{T})(0.10 \, \text{m})^2 \cos(60^\circ) = 1.0 \times 10^{-3} \, \text{T} \cdot \text{m}^2 \).

(c) \( \Phi = (0.20 \, \text{T})(0.10 \, \text{m})^2 \cos(90^\circ) = 0 \).

We can evaluate these results by comparing the calculated fluxes to the number of \( \vec{B} \)-field lines through the book’s cover area. Note that for the orientation of the book in (a) and (c), the \( \vec{B} \)-field lines are parallel to the book’s area and therefore do not go through it. That’s consistent with our mathematical result. The orientation for the book in (b) is such that some \( \vec{B} \)-field lines do pass through the book, which is also consistent with the nonzero mathematical result.

Try It Yourself: A circular ring of radius 0.60 m is placed in a 0.20-T uniform \( \vec{B} \)-field that points toward the top of the page. Determine the magnetic flux through the ring’s area when: (a) the plane of the ring is perpendicular to the surface of the page and its normal line points to the right; (b) the plane of the ring is perpendicular to the surface of the page and its normal line points toward the top of the page.

Answer: a) 0; b) 0.22 \( \text{T} \cdot \text{m}^2 \).

Review Question 18.2
You have a coil of wire connected to a power supply. You also have a plastic toy ring. How do you need to position the ring relative to the coil so that the magnetic flux through the ring is zero?

18.3 Direction of the Induced Current
Recall that in the experiments in Section 18.1, the galvanometer registered current in one direction for some of the experiments and in the opposite direction for others. We discovered the conditions under which a current can be induced in a wire loop, but can we also explain the direction of this current? That’s the goal of this section.

Fig. 18.10 shows the results of two experiments in which the number of \( \vec{B} \)-field lines through a wire coil’s area is changing. As the bar magnet moves closer to the coil in (a), the number of \( \vec{B} \)-field lines through the coil’s area increases (the magnetic flux through the coil’s...
area increases). As expected, there is a corresponding induced current. An arrow along the coil indicates the direction of this induced current as measured by a galvanometer.

We learned in Ch. 17 that electric currents produce a magnetic field. This means that the induced current in the coil also produces a magnetic field and magnetic flux through the coil. We call this magnetic field $\vec{B}_{\text{induced}}$. The direction of $\vec{B}_{\text{induced}}$ can be determined using the right hand rule for the fields. Notice that in the case shown in Fig. 18.10a, the flux through the coil due to the magnet (called $\vec{B}_{\text{external}}$) increases and the magnetic field due to the induced current $\vec{B}_{\text{induced}}$ points in the opposite direction.

In (b), the bar magnet is moving away from the coil. As a result the number of field lines through the coil’s area (and therefore the magnetic flux through it) is decreasing. Again, there is a corresponding induced current (see Fig. 18.10b). In this case however, $\vec{B}_{\text{induced}}$ due to the induced current points in the same direction as $\vec{B}_{\text{external}}$ due to the magnet. Can we find a pattern in these data?

Notice that in both cases $\vec{B}_{\text{induced}}$ points in the direction that diminishes the change in the external flux through the coil. In the first experiment the external flux through the coil was increasing. In that situation, $\vec{B}_{\text{induced}}$ pointed in the opposite direction to $\vec{B}_{\text{external}}$, as if it resisted the increase. In the second experiment, the external flux through the coil was decreasing and $\vec{B}_{\text{induced}}$ pointed in the same direction as $\vec{B}_{\text{external}}$, as if it resisted the decrease.

In both situations, the $\vec{B}_{\text{induced}}$ due to the induced current resisted the change in the original flux passing through the coil. Put another way, $\vec{B}_{\text{induced}}$ points in whatever direction is necessary to try to keep the magnetic flux through the coil’s area constant.

![Figure 18.10](image)

**Figure 18.10** How is changing $\vec{B}_{\text{external}}$ related to direction of $i_{\text{internal}}$ and $\vec{B}_{\text{internal}}$

Consider what would happen if the reverse occurred. Suppose an increasing external magnetic flux through the loop led to an induced magnetic field in the same direction as the external field. In that case, the magnetic flux due to the induced field would augment rather than reduce the total flux through the loop. This would cause an even greater induced current to flow,
which would cause yet a greater induced magnetic field and a steeper increase in magnetic flux. In other words, just by lightly pushing a bar magnet toward a loop of wire, you would cause a runaway induced current that would continually increase until the wire melted, which would violate the conservation of energy.

This pattern concerning the direction of the induced current was first developed in 1833 by the Russian physicist Heinrich Lenz.

**Lenz’s law** The induced current in a coil has a direction such that its $\vec{B}_{in}$-field opposes the change in the magnetic flux through the coil’s area produced by other objects. If the magnetic flux through the coil is increasing, the induced current’s $\vec{B}_{in}$-field contribution has a direction that leads to a decrease in the flux. If the magnetic flux through the coil is decreasing, the induced current’s $\vec{B}_{in}$-field has a direction that leads to an increase in the flux.

Lenz’s law lets us determine the direction of the induced current’s magnetic field $\vec{B}_{induced}$.

From there we can use the right hand rule for the $\vec{B}$-field to determine the direction of the induced current itself. This process is summarized in the following reasoning skill, which shows how to determine the direction of the induced current in a loop of wire. The loop of wire is placed in a region of uniform $\vec{B}$-field. The direction of the $\vec{B}$-field is perpendicular to the plane of the loop. The magnitude of the field is decreasing.
Reasoning Skill: Determine the direction of an induced current  

The magnetic flux through a coil can change because of a change in the external magnetic field, a change in the area of a coil, or a change in its orientation. Because of a flux change, a current is induced in a direction that can be determined as shown below.

1. Determine the initial external magnetic flux $\Phi_{ex,i}$ through the coil (represented below by the number and direction of the external magnetic field lines).

2. Determine the final external magnetic flux $\Phi_{ex,f}$ through the coil (represented below by the number and direction of the external magnetic field lines).

3. The induced flux tries to maintain the initial flux (the 4 up lines in this example).

4. Use the right hand rule for magnetic field to determine the direction of the induced current that will produce the $\vec{B}_{in}$ and the induced flux.

Conceptual Exercise 18.3 Direction of induced current in a coil  

A loop is placed in the plane of the page (Fig. 18.11a). A uniform $\vec{B}_{external}$ is produced in the region of the loop by a horseshoe magnet and points out of the page (represented by the dots). You pull the loop to the right at constant speed so that it leaves the magnetic field region. Which direction, if any, is the induced electric current in the loop as it is leaving the field region?
Sketch and Translate As the loop moves out of the magnetic field region, fewer $\vec{B}_{\text{ex}}$ lines pass up through its area. Thus, the external magnetic flux up out of the loop is decreasing (Fig. 18.11b).

Simplify and Diagram Thus, according to Lenz’s law the induced magnetic flux and induced magnetic field $\vec{B}_{\text{induced}}$ should point out of the paper as well (Fig. 18.11c), thus getting the net flux as the loop leaves the field closer to the initial flux while completely in the external field. The direction of the induced current that causes this induced magnetic field is determined using the right-hand rule for the $\vec{B}$-field; the induced electric current through the loop is counterclockwise. (Fig. 18.11c)

Try It Yourself: What direction would the induced current be if you pushed the coil back into the magnetic field region from the right side?
Answer: Clockwise.
**Eddy currents: an application of Lenz’s law**

If you were to hold a sheet of aluminum or copper between the poles of a strong horseshoe magnet, you would find that neither of the poles attracts the sheet. However, when you move the sheet out from between the poles of the magnet, especially if you pull it quickly, you encounter resistance. It is as if the magnet is trying to pull the sheet back. Let’s examine this phenomenon. Figure 18.12a shows a metal sheet between the poles of a magnet. When you pull the sheet to the right, the external magnetic flux through area 1 decreases (similar to the situation in the Conceptual Exercise 18.3 only there, the external field was up instead of down). This decrease in flux induces a current in the metal sheet around area 1, called an eddy current, that circles or curls clockwise in this region (see Fig.18.12b) and according to Lenz’s law produces an induced field that points in the same direction as the external field.

![Figure 18.12](image)

**Figure 18.12(a)** Pulling metal sheet to right induces eddy currents and an opposing magnetic force

At the same time, area 2 of the sheet is entering the magnetic field region, so the magnetic flux through that area is increasing. This change in magnetic flux induces a counterclockwise eddy current.

So, how do explain the force that points opposite the direction of the sheet’s motion? The left side of the eddy current in area 1, shown in Fig. 18.12b, is still in the magnetic field region. Remember from Ch. 17 that a magnetic field exerts a magnetic force on currents. Using the right hand rule for the magnetic force, we find that this force exerted by the magnet on the sheet points toward the left when the sheet is pulled to the right, in agreement with what was observed. Similarly, the magnet also exerts a force on the part of the eddy current in area 2 that is in the magnetic field region. The right hand rule for the magnetic force also determines that this force points to the left as well. Both of these forces exerted by the magnetic field on the eddy currents in the sheet point opposite the direction the sheet is moving, a sort of ‘braking’ force.

What would happen if you were to push the sheet to the left instead of pulling it to the right? Using Lenz’s law we find that the eddy currents reverse direction, and the magnetic forces exerted on them also reverse direction, again resulting in a magnetic braking effect.

The phenomenon of resistance to the motion of a non-magnetic metal material through a magnetic field is used for several practical purposes, such as reducing vibrations in sensitive scales, sorting coins in vending machines, and braking mechanisms for mass transit systems. The
braking system used in some cars, trains, and even amusement car rides consists of strong electromagnets with poles on either side of the turning metal wheels of the vehicle. When the electromagnet is turned on, magnetic forces are exerted by the electromagnet on the eddy currents in the wheels and oppose their motion. As the turning rate of the wheels slow, the eddy currents decrease, and therefore the braking forces decrease. Magnetic braking can bring a multi-ton vehicle traveling at speeds in excess of 100 mph to a smooth stop in a distance of about 200 m—for example, the huge Dreamworld’s Tower of Terror amusement car in Australia.

**Coin Sorter for Vending Machine**

According to legend, the first vending machine dates to 215 B.C., when the Greek mathematician Hero devised a machine for worshippers to deposit coins to get holy water. A coin dropped on a lever, which opened a valve letting the water flow. When the coin fell off the lever, water flow stopped. Today’s vending machines are equipped with coin sorters that use a magnetic field to induce eddy currents that slow each coin to a speed that is based on the coin’s metal type and size. Because the different coins exit the track at different speeds, they land in a bin that is specific for their type. Sensors detect where the coins land to determine how much the customer paid.

**Review question 18.3**

What difficulty would occur if the \( \vec{B} \)-field produced by the induced current enhanced the change in the external field rather than opposed the change? Give a specific example.

**18.4 Faraday’s Law of Electromagnetic Induction**

In the first two sections of this chapter, we found that when the magnetic flux through a coil’s area changes, there is an induced electric current in the coil. The flux depends on the magnitude of the \( \vec{B} \)-field, the area of the coil, and the orientation of the coil relative to the \( \vec{B} \)-field. Our next goal is to construct a quantitative version of this idea that will allow us to predict the magnitude of the induced current in a particular coil. (We already have a rule for predicting the direction of the induced current - Lenz’s law.) We start with the experiments in Observational Experiment Table 18.3 that use a magnet and different coils connected to a galvanometer to see how the time interval for a flux change through a coil and the number of turns in a coil affect the magnitude of the induced current produced by the flux change.
Observational Experiment Table 18.3 The effect of time interval and coil turns on induced current

<table>
<thead>
<tr>
<th>Observational Experiment</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rapidly move a magnet toward a coil and observe the galvanometer needle. Repeat the process, only move the magnet slowly.</td>
<td>The galvanometer registers a larger induced current when the magnet moves rapidly than when the magnet moves slowly toward the coil.</td>
</tr>
<tr>
<td>Rotate a magnet rapidly in front of a stationary coil. Repeat the process only rotate the magnet slowly.</td>
<td>The galvanometer registers a larger induced current when the magnet rotates rapidly than when it rotates slowly.</td>
</tr>
<tr>
<td>Use two coils, each with a different number of turns. Move the magnet toward the coil with the greater number of turns. Then move the magnet at the same speed toward the coil with fewer turns.</td>
<td>The galvanometer registers a larger induced current through the coil when it has a larger number of turns.</td>
</tr>
</tbody>
</table>

Patterns
• The speed at which the magnet moved or the coil rotated affected the induced current. The shorter the time interval for the change of the flux through the coil, the greater the induced current.
• The induced current is greater in a coil with a larger number of turns than for a coil with a smaller number of turns.

In the experiments in Observational Experiment Table 18.3, we found that the induced current was greater if the same change in magnetic flux through a coil occurred in a shorter time interval \( \Delta t \). Additionally, the induced current through the coil was greater in a coil with a larger number of turns.

**Tip!** Notice how even though we were investigating the effects of two different quantities on the magnitude of the induced current, we only changed one of them at a time so that we could investigate those effects separately.

**Faraday’s Law of Electromagnetic Induction**

How exactly does the magnitude of the induced current through a coil change as the magnetic flux through that coil’s area changes? Let’s connect a single circular loop with zero resistance in series to a 100Ω resistor (Fig. 18.13a). The loop is placed
between the poles of an electromagnet with its surface perpendicular to the magnetic field. An ammeter (not shown) measures the current through the loop and resistor. The upward, magnetic field (up is defined as the positive direction) decreases steadily for 2.0 s, after which the field and flux remains constant at a smaller positive value. Graphs for the magnetic flux-versus-time and the induced current-versus-time and are shown in Fig. 18.13b.

*Figure 18.13* An experiment to quantitatively relate $\varepsilon_{in}$ and $i_{in}$ to flux change

When the flux is *changing* at a constant rate, the current through the loop and resistor has a constant value. For example, for the first 2.0 s the *slope* of the magnetic flux-versus-time graph has a constant negative value of $-1.0 \text{T}\cdot\text{m}^2/\text{s}$, and the induced current has a constant positive value of $+0.010 \text{ A}$. If we replace the $100 - \Omega$ resistor with a $50 - \Omega$ resistor, the current-versus-time graph has the same shape but its magnitude during the first 2.0 s doubles to $+0.020 \text{ A}$. Thus, the same flux change produces different size currents in the loop depending on the resistive load. However, the product of the current and resistance for both situations has the same value:

$$(0.010 \text{ A})(100 \text{ }\Omega) = (0.020 \text{ A})(50 \text{ }\Omega) = 1.0 \text{ A} \cdot \text{ }\Omega = 1.0 \text{ V}.$$  

It almost seems that there must have been a 1.0-V battery in series with the resistor; but there wasn’t.

The changing flux through the loop caused the induced current. The *slope* of the flux-versus-time graph was:

$$\frac{\Delta \Phi}{\Delta t} = -1.0 \text{ T}\cdot\text{m}^2/\text{s} = -1.0 \left( \frac{N \cdot m^2}{C \cdot s} \right) = -1.0 \frac{\text{N}\cdot\text{m}}{\text{C} \cdot \text{s}} = -1.0 \frac{J}{C} = -1.0 \text{ V}.$$  

(We used the expression for the magnetic force $F_{\text{magnetic}} = q \nu B$ or $B = F_{\text{magnetic}} / q \nu$ to convert the magnetic field unit tesla (T) to other units). It seems that the rate of flux change is the “battery” that produces a 1.0-V emf that causes the electric current. You could apply Kirchhoff’s loop rule to the circuit shown in Fig. 17.13a. There is a $-1.0 \text{ V}$ change in potential across the loop and a $+1.0 \text{ V}$ potential change across the resistor (the signs could be reversed if went in opposite direction).
The magnitude of the emf provided by the loop depends only on the rate of change of flux through the loop (the slope of the flux-versus-time graph $\epsilon_{in} = \Delta \Phi / \Delta t$). When we repeat this experiment for a coil with $N$ loops, we find that the magnitude of the induced emf increases in proportion to the number $N$ of loops in the coil. Thus,

$$\epsilon_{in} = N \frac{\Delta \Phi}{\Delta t}.$$  \hspace{1cm} (18.2)

This expression is known as **Faraday’s law of electromagnetic induction**. However, the mathematical expression was actually developed by James Maxwell. His summary of the properties of electric and magnetic fields, and the relationships between them are known as Maxwell’s equations. Faraday’s law can be derived as a theoretical consequence of these equations.

**Faraday’s law for electromagnetic induction** The average magnitude of the induced emf $\epsilon_{in}$ in a coil with $N$ loops is the magnitude of the ratio of the magnetic flux change through the loop $\Delta \Phi$ and the time interval $\Delta t$ during which that flux change occurred multiplied by the number $N$ of loops:

$$\epsilon_{in} = N \left| \frac{\Delta \Phi}{\Delta t} \right| = N \left| \frac{\Delta [BA \cos \theta]}{\Delta t} \right|$$ \hspace{1cm} (18.3)

The direction of the current induced by this emf is determined using Lenz’s law. A minus sign is often placed in front of the $N$ to indicate that the induced emf opposes the change in magnetic flux.

**Tip!** The magnitude symbol $|$ means to find the absolute value of the quantity between the vertical lines.

Faraday realized that the phenomenon of electromagnetic induction has the same effect on devices attached to a coil as a battery does. This idea enabled him to build the first electric generator by using a coil that rotated between the poles of a magnet, resulting in an induced current through the coil.

**Quantitative Exercise 18.4 Hand-powered computer** To power computers in locations not reached by any power grid, engineers developed hand cranks that rotate a 100-turn coil inside a strong magnetic field region. In a quarter turn of the crank of one of these devices, the magnetic flux through the coil’s area changes from 0 to 0.10 T·m² in 0.50 s. What is the average magnitude of the emf induced in the coil during this quarter-turn?
Represent Mathematically Faraday’s law (Eq. 18.2) can be used to determine the average magnitude of the emf produced in the coil:

\[ \varepsilon_{av} = N \frac{\Delta \Phi}{\Delta t} = N \left( \frac{\Phi_f - \Phi_i}{t_f - t_i} \right) \]

Solve and Evaluate Inserting the appropriate values, we find:

\[ \varepsilon_{av} = (100) \left( \frac{(0.10 \, \text{T} \cdot \text{m}^2) - 0}{0.50 \, \text{s} - 0} \right) = 20 \, \text{V} \]

The magnitude of this emf is about what is required by laptop computers. However, the computer only works while the coil is turning. Laptops typically have a power requirement of about 50 watts (50 joule each second), which means considerable strength and endurance would be needed to keep the coil rotating. 

Try It Yourself: Determine the average emf produced in the coil if you turn the coil one-quarter turn in 1.0 s instead of in 0.50 s.

Answer: 10 V.

Review question 18.4
What physical quantities affect the magnitude of the induced emf produced in a coil of wire?

18.5 Skills for analyzing processes involving electromagnetic induction

Faraday’s law enables us to design and understand practical applications of electromagnetic induction. For example, to design an automobile ignition system that uses spark plugs, engineers must estimate how quickly the magnetic field through a coil must be reduced to zero to produce a large enough emf to ignite a spark plug. An engineer designing an electric generator will be interested in the rate at which the generator coil must turn relative to the \( \vec{B} \)-field to produce the desired induced emf. The general strategy for analyzing questions like these is described and illustrated in Example 18.5.

Example 18.5 Determining the \( \vec{B} \)-field produced by an electromagnet To determine the \( \vec{B} \)-field produced by an electromagnet, you use a 30-turn circular coil of radius 0.10-m (30-Ω resistance) that rests between the poles of the magnet and is connected to an ammeter.

When the electromagnet is switched off, the \( \vec{B} \)-field decreases to zero in 1.5 s. During this 1.5 s the ammeter measures a constant current of 180 mA. How can you use this information to determine the initial \( \vec{B} \)-field produced by electromagnet?
**Sketch and Translate**

- Create a labeled sketch of the process described in the problem. Show the initial and final situations to indicate the change in magnetic flux.
- Determine which physical quantity is changing (\( \vec{B} \), \( A \), or \( \theta \)) thus causing the magnetic flux to change.

**Simplify and Diagram**

- Decide what assumptions you are making; for example, did you assume that flux changes at a constant rate or that the magnetic field is uniform?
- If useful, draw a graph of the flux and the corresponding induced emf-versus-clock reading.
- If needed, use Lenz’s law to determine the direction of the induced current.

**Represent Mathematically**

- Apply Faraday’s law and indicate the flux quantity (\( \vec{B} \), \( A \), or the \( \cos \theta \)) that is changing.
- If needed use Ohm’s law and the loop rule to determine the induced current.

\[
\varepsilon_m = N \frac{\Phi_f - \Phi_i}{t_f - t_i} = N \frac{0 - B_i A \cos \theta}{\Delta t}
\]

The number of turns \( N \) and the angle \( \theta \) remain constant, the magnitude of the magnetic field changes.

Substitute \( \varepsilon_m = IR \) and \( A = \pi r^2 \) and solving for \( B_i \):

\[
B_i = \frac{IR\Delta t}{N\pi r^2 \cos \theta}
\]

**Solve and Evaluate**

- Use the mathematical representation to solve for the unknown.
- Evaluate the results—units, magnitude, and limiting cases.

The plane of the coil is perpendicular to the magnetic field lines, so \( \theta = 0 \) and \( \cos 0 = 1 \). Substituting other givens:

\[
B_i = \frac{IR\Delta t}{N\pi r^2} = \frac{0.18 \, \text{A} \times 30 \, \Omega \times 1.5 \, \text{s}}{30 \times \pi \times (0.1 \, \text{m})^2} = 8.6 \, \text{T}
\]

This is a very strong \( \vec{B} \)-field but possible with modern superconducting electromagnets. Let’s check the units:

\[
\frac{A \cdot \Omega \cdot \text{s}}{\text{m}^2} = \frac{V \cdot \text{s}}{\text{C} \cdot \text{m}^2} = \frac{J \cdot \text{s}}{\text{C} \cdot \text{m}^2} = \frac{N \cdot \text{m} \cdot \text{s}}{C \cdot \text{m}^2} = \frac{N}{C \cdot (\text{m} / \text{s})} = \text{T}
\]

The units match. As a limiting case, a coil with few turns would require a larger \( \vec{B} \)-field to induce the same current. Also, if the resistance of the circuit is large, the same \( \vec{B} \)-field change induces a smaller current.

**Try It Yourself:** Determine the current in the loop if the plane of the loop is in the same direction as the magnetic field. Everything else is the same.

**Answer:** Zero.
In the following three sections, we consider practical applications of electromagnetic induction that involve a change in: 1) the magnitude of the $\vec{B}$-field, 2) the area of the loop or coil, or 3) the orientation of the coil relative to the $\vec{B}$-field. All of these processes involve the same basic idea: a changing magnetic flux through a coil’s or single loop’s area are accompanied by an induced emf through the coil. In turn, this emf induces an electric current in the coil.

**Review Question 18.5**

In the last example, why did we assume that the $\vec{B}$-field was uniform?

### 18.6 Changing $\vec{B}$-field magnitude and corresponding induced emf

We learned that a current is induced in a coil (or single loop, or electrically conductive region in the case of eddy currents) when the magnetic flux through the coil’s area changes. In this section, we consider examples where the flux change is due to a change in the magnitude of the $\vec{B}$-field throughout the coil’s area. This is the case in transcranial magnetic stimulation (TMS), which we investigated qualitatively earlier in this chapter.

**Example 18.6 Transcranial magnetic stimulation** The $\vec{B}$-field from a TMS coil increases from 0 T to 0.2 T in 0.002 s. The $\vec{B}$-field lines pass through the scalp into a small region of the brain, inducing a small circular current through the conductive brain tissue in the plane perpendicular to the field lines. Assume that the radius of the circular current in the brain is 0.0030 m and that the tissue in this circular region has an equivalent resistance of 0.010 $\Omega$. In which direction is the induced current, and what is the magnitude of the induced electric current around this circular region of brain tissue?

**Sketch and Translate** A sketch of the situation shows a small coil on the top of the scalp and a small circular disk inside the brain tissue through which the changing magnetic field flows (Fig. 18.14a). The change in magnetic flux through this disc is caused by the increasing $\vec{B}$-field produced by the TMS coil (called $\vec{B}_{\text{coil}}$). This change in flux causes an induced emf, which produces an induced current in the brain tissue. The direction of this current can be determined using Lenz’s law.
Figure 18.14(a) Transcranial magnetic stimulation, changing current in the coil induces a current loop in the brain.

Simplify and Diagram Assume that \( \mathbf{B}_{\text{coil}} \) throughout the disc of brain tissue is uniform and increases at a constant rate. Model the disc-like region as a single-turn coil. Viewed from above, \( \mathbf{B}_{\text{coil}} \) points into the page (marked as X’s in Fig. 18.14b). Since the number of \( \mathbf{B} \)-field lines is increasing into the page, the \( \mathbf{B} \)-field produced by the induced current will point out of the page (shown as dots.) Using the right-hand rule for the \( \mathbf{B} \)-field, we find that the direction of the induced current is counterclockwise.

Figure 18.14(b)

Represent Mathematically To find the magnitude of the induced emf, use Faraday’s law:

\[
\varepsilon_{\text{in}} = N \frac{\Phi_f - \Phi_i}{t_f - t_i}
\]

where the magnetic flux through the loop at a specific clock reading is \( \Phi = BA \cos \theta \). The area \( A \) of the loop and the orientation angle \( \theta \) between the loop’s normal line and the \( \mathbf{B} \)-field are constant, so:

\[
\varepsilon_{\text{in}} = N \frac{B_f A \cos \theta - B_i A \cos \theta}{t_f - t_i} = NA \cos \theta \left| \frac{B_f - B_i}{t_f - t_i} \right|
\]

Using our understanding of electric circuits we relate this induced emf to the resulting induced current:
\[ i_{in} = \frac{\varepsilon_{in}}{R} \]

**Solve and Evaluate** Combine these two equations and solve for the induced current:

\[
i_{in} = \frac{1}{R} \varepsilon_{in} = \frac{1}{R} \left( NA \cos \theta \left| \frac{B_f - B_i}{t_f - t_i} \right| \right) = \frac{N \pi r^2 \cos \theta}{R} \left| \frac{B_f - B_i}{t_f - t_i} \right|
\]

\[
= \left( \frac{1}{0.010 \, \Omega} \right) \pi \left( 0.0030 \, \text{m} \right)^2 \left( \frac{1}{0.2 \, \text{T} - 0} \right) = 0.094 \, \text{A}
\]

Note that the normal line to the loop’s area is parallel to the magnetic field (Fig. 18.14a); therefore \( \cos \theta = \cos (0^\circ) = 1 \). This is a reasonable current and could affect brain function in that region of the brain. We found earlier that the induced current is counterclockwise as seen looking down from above in Fig. 18.14b.

**Try It Yourself:** A circular coil of radius 0.020 m with 200 turns lies so that its area is parallel to this page. A bar magnet above the coil is oriented perpendicular to the coil’s area, its north pole facing towards the coil. You quickly (in 0.050 s) move the bar magnet sideways away from the coil to a location far away. During this 0.050 s, the magnitude of the \( \vec{B} \)-field throughout the coil’s area changes from 0.40 T to 0 T. Determine the average magnitude of the induced emf around the coil while the bar magnet is being moved away.

**Answer:** 2.0 V.

There are many applications for TMS. For example, repetitive TMS pulses (called rTMS) can treat mood disorders. High-rate TMS applied to the left dorsolateral prefrontal cortex could replace electroconvulsive shock therapy in the treatment of depression. TMS can also help identify regions of the brain used for different purposes. For example, TMS stimulation of one region of the cortex in blind patients induced errors in Braille reading tasks whereas stimulation of other regions did not. In another study, subjects were shown brief, randomly generated letters on a visual monitor. When TMS was delivered 80–100 ms after the visual stimulus to another region of the cortex, subjects reported seeing blurred images or nothing at all, indicating that during that time interval, important visual processing had been interrupted in that part of the cortex. Similarly, TMS impulses to another part of the cortex delayed reaction time tasks. TMS studies have helped medical researchers understand the processes involved in neural repair, learning, and memory.

**Review Question 18.6**

How can a coil of wire, a battery, and a switch placed outside a patient’s head produce electric currents in the brain?
18.7 Changing area and corresponding induced emf

When located in a region with non-zero $\vec{B}$-field, a change in a coil’s (or single loop’s or conducting region’s) area also results in a change in the magnetic flux through that area. There then will be a corresponding induced emf producing an induced electric current circling the area.

**Example 18.7 Lighting a bulb** A 10-$\Omega$ light bulb is connected between the ends of two parallel conducting rails that are separated by 1.2 m (Fig. 18.15a.) A metal rod is pulled along the rails so that it moves to the right at a constant speed of 6.0 m/s. The two rails, the light bulb, its connecting wires, and the rod make a complete circuit rectangular loop. A uniform 0.20-T magnitude $\vec{B}$-field points downward, perpendicular to the loop’s area. Determine the direction of the induced current in the loop, the magnitude of the induced emf, and the magnitude of the current through the light bulb.

![Figure 18.15(a) Changing area causes induced emf and current](image)

**Sketch and Translate** A sketch of the situation shows the rails and rod considered as a complete circuit loop. Choose the normal line for the loop’s area to point upward. The loop’s area increases as the rod moves away from the bulb. Because of this, the magnitude of the magnetic flux through the loop’s area is increasing as the rod moves to the right (Fig. 18.15b.)

![Figure 18.15(b)](image)

**Simplify and Diagram** Assume that the rails, rod, and connecting wires have zero resistance. The induced $\vec{B}$-field due to the loop’s induced current should point upward, resisting the change in the downward increasing magnetic flux through the loop’s area. Using the right-hand rule for the $\vec{B}$-field, we find that the direction of the induced current is counterclockwise.
Represent Mathematically To find the magnitude of the induced emf, use Faraday’s law:

\[ \mathcal{E}_m = N \frac{\Phi_f - \Phi_i}{t_f - t_i} \]

The angle between the loop’s normal line and the $\vec{B}$-field is 180°. The magnitude of the $\vec{B}$-field is constant. Therefore:

\[ \mathcal{E}_m = (1) \frac{BA_f \cos(180°) - BA_i \cos(180°)}{t_f - t_i} = B \frac{A_f - A_i}{t_f - t_i}. \]

The area $A$ of the loop at a particular clock reading equals the length $L$ of the sliding rod times the $x$-coordinate of the sliding rod (the origin of the $x$-axis is placed at the bulb.) Therefore:

\[ \mathcal{E}_m = B \frac{Lx_f - Lx_i}{t_f - t_i} = BL \frac{x_f - x_i}{t_f - t_i} = BLv. \quad (18.3) \]

The quantity inside the absolute value is the $x$-component of the rod’s velocity, the absolute value of which is the rod’s speed $v$. Using Ohm’s law, the induced current depends on the induced emf and the bulb resistance:

\[ i_m = \frac{\mathcal{E}_m}{R} \]

Solve and Evaluate The magnitude of the induced emf around the loop is:

\[ \mathcal{E}_m = BLv = (0.20 \, \text{T})(1.2 \, \text{m})(6.0 \, \text{m/s}) = 0.72 \, \text{V}. \]

The current through the bulb is:

\[ i_m = \frac{0.72 \, \text{V}}{10 \, \Omega} = 0.072 \, \text{A}. \]

The lamp should glow, but just barely since its power output is just:

\[ P = i_m^2 R = (0.072 \, \text{A})^2 (10 \, \Omega) = 0.052 \, \text{W}. \]

Try It Yourself: Suppose the rod in the last example moves at the same speed but in the opposite direction so that the loop’s area decreases. Determine the magnitude of the induced emf, the magnitude of current through the bulb, and the direction of the current.

Answer: 0.72 V, 0.072 A, and clockwise.

Limitless electric energy?

We just found that the induced emf depended on the speed with which a metal rod is pulled along the rails. By accelerating the rod to a speed of our choosing, we could induce whatever emf we desired. Once this speed is reached, would this method maintain the current indefinitely, or is there a catch?
The moving rod resulted in an emf and an induced current through the rails, bulb, and rod. However, if we use the right hand rule for magnetic force, we find that the magnetic field exerts a force on the induced current in the rod toward the left, causing it to slow down. From an energy perspective, the kinetic energy of the rod is being transformed into the light and thermal energy in the bulb. In order to keep the rod moving at constant speed, some other object needs to exert a force on it continuously pulling it to the right to do positive work on it.

**Motional emf**

The emf produced in the last example by the rod moving away from the bulb is an example of what is sometimes called *motional emf*; it results from the motion of an object through a magnetic field region. This emf was explained using the idea of electromagnetic induction. Is it possible to understand it just in terms of magnetic forces? When an electrically charged object with charge $q$ moves within a region with nonzero $\vec{B}$-field, the field exerts a magnetic force on it ($F_{\vec{B} on q} = |q|vB\sin\theta$). Consider the system shown in Fig. 18.16a with the rod sliding at velocity $\vec{v}$ along the rails. The external magnetic field $\vec{B}_{\text{ext}}$ points into the page. Inside the rod, there are fixed positively charged ions and negatively charged free electrons. When the rod slides to the right, all of its charged particles move with it. According to right-hand rule for forces, the external magnetic field exerts a magnetic force on negative electrons toward end I. The positive charges cannot move inside the rod, but the free electrons can. The electrons accumulate at end I, leaving end II with a deficiency of electrons (a net positive charge). The ends of the rod become charged as shown in Fig. 18.16b.

![Figure 18.16(a)](image1)

![Figure 18.16(b)](image2)

![Figure 18.16(c)](image3)

These separated charges create an electric field $\vec{E}$ in the rod that exerts a force of magnitude $F_{\vec{E} on q} = qE$ on other electrons in the rod; the electric field exerts a force on negative electrons toward II (Fig. 18.16c) opposite the direction of the magnetic force. An electric
potential difference is produced between points I and II and depends on the magnitude of the electric field \( \vec{E} \) in rod I-II and the distance \( L \) between I and II:

\[
\varepsilon = \Delta V_{II} = EL \quad \text{(17.xx)}
\]

When the magnitude of the electric force equals the magnitude of the magnetic force, the accumulation of opposite electric charge at the ends of the rod ceases. Then,

\[
qvB = qE \quad \text{or} \quad E = vB
\]

Thus, the magnitude of the potential difference between I and II equals the motional emf:

\[
\varepsilon_{\text{motional emf}} = \Delta V_{II} = EL = vBL \quad \text{(18.4)}
\]

The motional emf is caused by the motion of the electric charges in the metal rod in the magnetic field. The above expression for motional emf is the same expression as we derived in Example 18.7 using Faraday’s law. Thus, for problems involving conducting objects moving in a magnetic field, we can use either Faraday’s law or the motional emf expression to determine the emf—either method will provide the same result. In Section 18.1, we derived emf qualitatively; here we have a quantitative description.

Review question 18.7

Suppose the rod in Example 18.7 was 1/3 the length and the magnetic field was 4/5 the magnitude. How fast would the rod need to move to induce the same emf? Would the current induced in this case be the same as for Example 18.7? Explain.

18.8 Changing orientation and corresponding induced emf

In the previous two sections, we investigated situations where emf was induced when the magnitude of the \( \vec{B} \)-field changed or when the area of a loop (or coil or conducting region) within the \( \vec{B} \)-field region changed. In this section we investigate what happens when the orientation of the loop relative to the direction of the \( \vec{B} \)-field changes. This has many practical applications with one of the most important being the electric generator.

The electric generator

The U.S. transforms on average 1000 J per second of electric potential energy per person into other less useful forms every second of every day. This is equivalent to keeping ten 100 W light bulbs continuously lit per person. This also amounts to about 24 percent of the electric potential energy transformed worldwide. Electric generators make this electrical potential energy available by transforming mechanical energy (such as water rushing through a hydroelectric dam) into this electric energy.

To understand how an electric generator works, consider a very simple device that consists of a loop of wire attached to a turbine (a propeller-like object that can rotate). The loop is positioned between the poles of an electromagnet that produces a steady uniform \( \vec{B} \)-field. Water
The water converts to steam, which strikes the blades of the turbine, causing the turbine to rotate. A loop of wire attached to the turbine rotates in the $\vec{B}$-field region. When the loop’s surface is perpendicular to the $\vec{B}$-field, the magnetic flux through the loop’s area is maximum. One quarter turn later, the $\vec{B}$-field lines are parallel to the loop’s area and the flux through it is zero. After another quarter turn, the flux is again maximum, but negative in value since the orientation of the loop’s area is opposite what it was originally. This changing magnetic flux through the loop’s area has a corresponding induced emf which produces a current that changes direction each time the loop rotates one half turn. This is what is known as alternating current (AC).

Figure 18.17 A homemade version of electric power plant

A coal-fired power plant works in a way similar to this device. In this type of power plant, coal burns to heat water, converting it to steam. The high-pressure steam pushes against turbine blades, causing the turbine and an attached wire coil to rotate in a strong $\vec{B}$-field. This results in an emf that helps drive the electric power grid.

How can we determine an expression for the emf produced by an electric generator? Consider the changing magnetic flux through a loop’s area as it rotates with constant rotational speed $\omega$ in a constant uniform $\vec{B}$-field (Fig. 18.18).

If there is an angle $\theta$ between the loop’s normal line and the $\vec{B}$-field, then the flux through the loop’s area is:

$$\Phi = BA \cos \theta.$$
Since the loop is rotating, $\theta$ is continuously changing. Since the loop is rotating with zero rotational acceleration ($\alpha = 0$), we can describe the motion with rotational kinematics (see Chapter 8):

$$\theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 = \theta_0 + \omega t + \frac{1}{2} (0)t^2 = \theta_0 + \omega t$$

If we define the initial orientation $\theta_0$ to be zero, then:

$$\theta = \omega t,$$

where $\omega$ is the constant rotational speed of the loop. This means that the magnetic flux $\Phi$ through the loop’s area as a function of time $t$ is:

$$\Phi = BA \cos(\omega t).$$

A side view of the rotating loop is shown in Fig. 18.19a. A graph of the magnetic flux through the loop’s area as a function of time is shown in Fig. 18.19b.

![Figure 18.19](image)

**Figure 18.19** (a) As the coil turns (b) the flux through it changes as does (c) the emf across it.

According to Faraday’s law, the induced emf around the coil is:

$$\varepsilon_{\text{in}} = N \left| \frac{\Delta \Phi}{\Delta t} \right| = N \left| \frac{\Phi_f - \Phi_i}{t_f - t_i} \right| \quad (18.2)$$

Since $\Phi$ is continually changing, the above should be rewritten using a calculus derivative:

$$\varepsilon_{\text{in}} = N \left| \frac{d \Phi}{dt} \right| = N \left| \frac{d(BA \cos(\omega t))}{dt} \right| = NBA \omega \sin(\omega t) \quad (18.3)$$

where $N$ is the number of turns in the coil rotating between the poles of the magnet. If calculus is not used in your course, you will have to just accept the above.

A graph of the induced emf as a function of clock reading is shown in Fig. 18.19c. If you look carefully at Fig. 18.19b and c, you see a pattern. The value of $\varepsilon_{\text{in}}$ at a particular clock reading equals the negative value of the slope of the $\Phi$ vs. $t$ graph at that same clock reading.
This makes sense since the induced emf is related to the rate of change of the magnetic flux through the loop’s area. Slopes represent exactly that, rates of change.

Electric power plants in the U.S. produce an emf with frequency $f$ equal to 60 Hz. This corresponds to a generator coil rotating with a rotational speed:

$$\omega = 2\pi f = 2\pi (60 \text{ Hz}) = 120\pi \text{ rad/s}.$$ 

They can produce a peak (maximum) emf as high as 20 kV. The peak emf produced by a generator occurs when $\sin(\omega t) = 1$. At those times,

$$e_{\text{in max}} = NBA\omega \quad (18.4)$$

**Quantitative Exercise 18.8 Bicycle light generator** The label on the Schmidt E6 bicycle dynamo headlight indicates that the light has a power output of 3 W and a peak emf of 6 V. The generator (also called a dynamo) for the light bulb has a cylindrical hub that rubs against the edge of the bike tire, causing a coil inside the generator to rotate (Fig. 18.20). When the bicycle is traveling at a speed of 12 mph, the coil rotates with frequency of $800 \text{ Hz}$ ($800 \text{ revolutions per second}$). The $\vec{B}$-field in the vicinity of the coil is uniform and has a magnitude of 0.10 T. The coil is a rectangle with dimensions 1.0 cm x 3.0 cm. Without taking the light apart, determine how many turns there are in the generator coil.

Figure 18.20 Bicycle tire causes coil to turn in magnetic field thus generating electric current

**Represent Mathematically** The number of turns in the coil is related to the maximum emf the generator can produce (Eq. 18.4):

$$e_{\text{in max}} = NBA\omega = NBA\left(2\pi f\right)$$

**Solve and Evaluate** Solving for $N$ and inserting the appropriate values:

$$N = \frac{e_{\text{in max}}}{2\pi fBA} = \frac{6.0 \text{ V}}{2\pi (800 \text{ Hz})(0.10 \text{ T})(0.01 \text{ m} \times 0.03 \text{ m})} = 40.$$
A generator coil with this number of turns is reasonable. Let’s check limiting cases. If either the magnetic field, the loop area, or the frequency is larger, then fewer turns are needed for the peak emf to be 6.0 V, all of which are reasonable.

**Try It Yourself:** While riding your bike up a hill, you pedal harder; however, your bike speed reduces from 12 mph to 6 mph. How would these conditions affect the emf produced by the bicycle light generator in the last example?
**Answer:** The emf would be 3.0 V, since the loop turning frequency would decrease to half the previous value.

**Review Question 18.8**
How does the law of electromagnetic induction explain why there is an induced emf in a rotating generator coil?

### 18.9 Transformers

Another useful application of electromagnetic induction is the transformer, which converts an alternating emf from a lower to a higher emf, or vice versa. For example, if you drive a car that uses spark plugs for ignition, a transformer is one part of an electric system that converts the 12-V potential difference of the car battery to the 20,000-V potential difference needed to produce a spark in the engine’s cylinder.

A transformer consists of two coils each wrapped around a magnetic core. (Fig. 18.21) The core confines the magnetic field produced by electric current in one coil so that it passes through the second coil. An alternating emf across the primary coil is converted into a larger or smaller alternating emf across the secondary coil, depending on the number of loops in each coil.

![Figure 18.21](image)

*Figure 18.21* The transformer changes the input/output emf depending on ratio of turns

Transformers are used in many devices, such as cell phones, digital cameras, ink jet printers, electric razors, electric drills, and metal detectors. They are also essential for the transmission of electric energy from a power plant to your house. The rate of this electric energy transmission is proportional to the product of the emf across the power lines and the electric current in the lines. If the emf is low, then to have considerable power transmission requires
considerable electric current. The lines however have electric resistance that leads to considerable thermal energy production ($I^2R$ loses). Thus, the transmission of electric energy is done at high peak emf (about 20,000 V) and low current causing reduced $I^2R$ loses. Transformers then reduce this peak emf to about 170 V for use in your home. How does a transformer change the peak voltage?

Suppose there is an alternating current in the primary coil, the coil connected to an external power supply. The secondary coil is connected to an electrical device, but this device requires an emf that is different than the external power supply produces. The alternating current in the primary coil produces a $\vec{B}$-field within the transformer core. Since the current is continuously changing, the magnetic flux through the primary coil’s area is also changing. Thus, an emf is induced in the primary coil. According to Faraday’s law it equals:

$$\varepsilon_p = N_p \frac{\Delta \Phi_p}{\Delta t}$$

where the ‘$p$’ subscript refers to the primary coil.

In an efficient transformer, nearly all of the $\vec{B}$-field lines passing through the primary coil’s area also pass through the secondary coil’s area (the ferromagnetic core confines the $\vec{B}$-field.) As a result, there is a changing magnetic flux through the secondary coil’s area, and also a corresponding emf produced in it. This emf equals (again, by Faraday’s law):

$$\varepsilon_s = N_s \frac{\Delta \Phi_s}{\Delta t}$$

If the transformer is perfectly efficient, then the rate of change of magnetic flux through both coils is equal:

$$\frac{\Delta \Phi_p}{\Delta t} = \frac{\Delta \Phi_s}{\Delta t}.$$

Using the results from Faraday’s law:

$$\frac{\varepsilon_p}{N_p} = \frac{\varepsilon_s}{N_s}$$

$$\Rightarrow \varepsilon_s = \frac{N_s}{N_p} \varepsilon_p \quad (18.5)$$

We see that the emf in the secondary coil can be substantially larger or smaller than the emf in the primary coil depending on the number of turns in each coil. Engineers use this result to design transformers for specific purposes. For example, a so-called step-down transformer can convert the 120-V alternating emf from a wall socket to a 9-V alternating emf, which is then converted to DC to power a laptop computer.
Quantitative Exercise 18.9 Transformer for laptop

Suppose that your laptop requires a 24-V emf to function. What should be the ratio of the primary coil turns to secondary coil turns if this transformer is to be plugged into a standard house AC outlet (effectively a 120-V emf.)

*Represent Mathematically* The ratio we are looking for is related to the coil emfs by Eq. (18.5):

\[ \varepsilon_s = \frac{N_s}{N_p} \varepsilon_p. \]

*Solve and Evaluate* Solving for the ratio and inserting the appropriate values:

\[ \frac{N_p}{N_s} = \frac{\varepsilon_p}{\varepsilon_s} = \frac{120 \text{ V}}{24 \text{ V}} = 5. \]

The primary coil needs to have five times the number of turns as the secondary coil. This is an example of a step-down transformer since the resulting secondary coil emf is lower than the primary coil emf.

A 20,000-V spark from a 12-V battery!

How can a transformer be used to allow a 12-V car battery to produce a 20,000-V spark that ignites fuel in the cylinders of the car’s engine (Fig. 18.22)? The battery is connected to the primary coil of a transformer resulting in a steady current through it. An electronic switching system in the circuit can open the circuit stopping the current in the primary coil in a fraction of a millisecond. This causes an abrupt change in the magnetic flux through the primary coil of the transformer. This change in the magnetic flux leads to an induced emf in the secondary coil. The secondary coil is attached to a spark plug that has a gap between two conducting electrodes. When the potential difference across the gap becomes sufficiently high, the air between them ionizes. When the ionized atoms recombine, the energy is released in the form of light – we see a spark. This spark ignites the gasoline.

![Figure 18.22 Getting a 20,000 V spark from a 12-V car battery](image)

The induced emf in the secondary coil is much greater than the 12 V in the primary coil for three reasons. First, the secondary coil has many more turns than the primary coil \((N_s \gg N_p)\).
Second, the magnetic flux through the primary coil’s area decreases very quickly (the $\Delta t$ in the denominator of Faraday’s law is very small) resulting in a large induced emf $\varepsilon_p$ to which $\varepsilon_i$ is proportional (see Eq. 18.5). Third, the ferromagnetic core (usually iron) passing through the two coils significantly increases the $\vec{B}$-field within it (Section 17.10.) For these three reasons, it’s possible for $\varepsilon_i \gg \varepsilon_p$; the 12-V car battery produces a 20,000-V potential difference across the electrodes of a spark plug.

**Review Question 18.9**
How does a transformer achieve different induced emfs on its primary and secondary coils?

### 18.10 Mechanisms Explaining Electromagnetic Induction

Faraday’s law describes how a changing magnetic flux through a wire loop is related to an induced emf; but it does not explain how the emf is induced. In other words, it does not explain the mechanism for the phenomenon of electromagnetic induction. At the beginning of this chapter, we devised a mechanism that explained some of the experiments related to the electromagnetic induction.

This mechanism is based on the idea that when an electrically charged particle moves in a magnetic field, the field can exert a force on it ($F_{\vec{B}\cdot\vec{r}} = qv\sin B \sin \theta$). In some electromagnetic induction experiments, a conductor (for example, a metal rod) with electrons inside moves in a magnetic field. The magnetic force causes a charge separation and a motional emf across the rod. If the rod is part of a closed conducting loop, the induced current will appear in the loop. Thus, we can use the magnetic force as one mechanism to explain electromagnetic induction. However, you now know that conductive objects do not have to be in motion in order for a current to be induced. Thus, to explain the phenomenon of the electromagnetic induction we need a mechanism that accounts for the cases when there is no motion.

**A changing $\vec{B}$-field has a corresponding $\vec{E}$-field**

Consider a stationary circular loop of wire placed in a region with increasing $\vec{B}$-field (Fig. 18.23a). We’ve learned that a changing magnetic flux through the loop’s area induces an electric current in the loop. Since the loop is not moving, there is no net magnetic force exerted on free electrons in the wire. To account for the production of the induced current in the case of no motion, physicists assume that whenever you have a changing magnetic field in a region of space, there is always a corresponding electric field in the same region (see the experiment shown in Fig. 18.23b). This electric field exerts an electric force on the free electrons inside the loop’s wire, thus causing the induced current. Unlike previous situations involving electric field that we encountered, this electric field is not produced by charge separation.
Since there is an induced current in the loop, there must be an emf, but this emf is very different than the emf produced by the chemical reactions inside a battery. In that case we are able to identify the location of the emf. In this case, we can’t. The electric field that is driving the current is everywhere in the wire, so the emf is actually distributed throughout the loop. You might visualize it as an electric field ‘gear’ with its teeth hooked into the electrons in the wire loop, pushing the free electrons along the wire at every point. This $\vec{E}$-field is represented by field lines that do not have beginnings nor ends—they close on themselves, very similar to the way $\vec{B}$-field lines do. This electric field is what produces the emf described quantitatively by Faraday’s law.

Consider the electromagnet shown in Fig. 18.24. An alternating emf powers the circuit causing an electric current that produces a magnetic field that changes direction and magnitude with time. If we place a coil attached to a light bulb stationary above the electromagnet, the bulb glows. We can use the electric field idea to explain this phenomenon. When the magnetic field of the electromagnet changes, there is an electric field which in turn produces the current in the wire that makes the light bulb glow. The work that this electric field does on the moving charged objects in the wire is transformed into as light and thermal energy of the light bulb. We will see in Chapter 24 how this electric field helps us explain the propagation of radio waves.
What do we now know about electricity and magnetism?

In the last five chapters, we learned a great deal about electric and magnetic phenomena, including new physical quantities and laws relating these quantities to each other. In Chapters 14 and 15 we learned about electrically charged objects that interact via electrostatic (Coulomb’s) forces. Stationary electrically charged objects produce electric fields. Electric field lines start on positive charges and end on negative (Fig. 18.25a). In Chapter 17, we learned that moving electrically charged objects and permanent magnets interact via magnetic forces and produce magnetic fields. The field lines of magnetic fields do not have a beginning or end (Fig. 18.25b); there are no magnetic charges (magnetic monopoles) on which they start or end.

In Chapter 16 we learned that electric fields cause electrically charged particles inside metal wires to move in a coordinated way, and in Chapter 17 that electric current produces magnetic fields (Fig. 18.25c). In this chapter, we learned the phenomenon of electromagnetic induction and its explanation: a changing magnetic field is accompanied by a corresponding electric field (Fig. 18.25d). However, this new electric field is not electrostatic—it is not created by electric charges and thus its field lines do not have a beginning and an end.

Except for the lack of magnetic charges, there is symmetry in the behavior of electric and magnetic fields; therefore, we can pose the following question. If in the area where magnetic field is changing there is an electric field, then why isn’t there a magnetic field in the area where electric field is changing? (Fig. 18.25e) In 1862, James Clerk Maxwell summarized physics knowledge about electricity and magnetism. The question above motivated his hypothesis that in a region where there is a changing electric field there must be a magnetic field, just as in the area with a changing magnetic field there is an electric field. Maxwell’s hypothesis launched a unified theory of electricity and magnetism that later also included heat and light. We will learn about this theory in Chapter 24.

Review question 18.10

Explain how (a) an electric current is produced when a single wire loop moves through a magnetic field and (b) an electric current is produced when an external magnetic flux changes through a closed loop of wire.
## Summary

<table>
<thead>
<tr>
<th>Word Representation</th>
<th>Pictorial or Physical Representation</th>
<th>Mathematical Representation</th>
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<tr>
<td>The magnetic flux through a loop’s area depends on the size of the area, the $\vec{B}$-field magnitude, and the orientation of the loop relative to the $\vec{B}$-field.</td>
<td><img src="image1.png" alt="Magnetic Flux Diagram" /></td>
<td>$\Phi = BA \cos \theta$</td>
</tr>
<tr>
<td><strong>Electromagnetic induction</strong> Electric current is induced in a wire loop, coil, or conductive region when the magnetic flux through that loop changes.</td>
<td><img src="image2.png" alt="Electromagnetic Induction Diagram" /></td>
<td>$\varepsilon_n = N \frac{\Delta \Phi}{\Delta t} = N \frac{\Phi_f - \Phi_i}{t_f - t_i}$ (18.2)</td>
</tr>
<tr>
<td><strong>Lenz’s law</strong> When current is induced, its direction is such that the $\vec{B}$-field it produces opposes the change in the magnetic flux through the loop.</td>
<td><img src="image3.png" alt="Lenz’s Law Diagram" /></td>
<td></td>
</tr>
<tr>
<td>An electric generator produces an emf by rotating a coil within a region of strong $\vec{B}$-field, an important application of electromagnetic induction.</td>
<td><img src="image4.png" alt="Electric Generator Diagram" /></td>
<td>$\varepsilon_n = NBA\omega \sin(\omega t)$ (18.3)</td>
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<td>Transformers are electrical devices that allow the conversion of an alternating emf to a higher or lower emf.</td>
<td><img src="image5.png" alt="Transformer Diagram" /></td>
<td>$\varepsilon_s = \frac{N_s}{N_p} \varepsilon_r$ (18.5)</td>
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