West Windsor-Plainsboro Regional School District
AP Statistics

Unit 1: Exploring One-Variable Data

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

This unit introduces students to data and the vocabulary of statistics. Students also learn to talk about data in real-world contexts. Variability in data may seem to suggest certain conclusions about the data distribution, but not all variation is meaningful. Statistics allows us to develop shared understandings of uncertainty and variation. Students will define and represent categorical and quantitative variables, describe and compare distributions of one-variable data, and interpret statistical calculations to assess claims about individual data points or samples. Students will also begin to apply the normal distribution model as an introduction to how theoretical models for populations can be used to describe some distributions of sample data.

Recommended Pacing

16 days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with mathematics
6	Attend to precision

Standard: S-ID.A Summarize, represent, and interpret data on a single count or measurement variable

CPI #	Cumulative Progress Indicator (CPI)
1	Summarize, represent, and interpret data on a single count or measurement variable
2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Technical Reading

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- Statistics relates to the art of data.
- There are data analysis strategies to explore categorical and quantitative data.
- You can display quantitative data graphically as well as describe it with numbers.
- Percentiles are used to identify where a value is located in a distribution.
- Density curves is a way of modeling distributions with a smooth curve.

Unit Essential Questions

- When do we use categorical and when do we use quantitative variables?
- What makes a misleading graph recognizable?
- How do you describe a distribution?
- What is a cumulative relative graph used for?
- What are the properties of a Normal Distribution?

Objectives

Students will know:

- Vocabulary: statistics, individual, variable, categorical variable, quantitative variable, distribution, frequency table, relative frequency table, bar graph, pie chart, two-way table, marginal relative frequency, joint relative frequency, conditional relative frequency, side-by-side bar graph, segmented bar graph, association, dotplot, symmetric distribution, skewed distribution, stemplot, histogram, mean, resistant, median, range, standard deviation, quartiles, IQR, five-number summary, boxplot, percentile, cumulative relative frequency graph, standardized score, density curve, standard normal distribution, normal probability plot
- How to identify the individuals and variables in a set of data
- How to identify what makes some graphs of categorical data misleading
- How to identify the shape of a distribution from a graph
- How to identify outliers using the 1.5 X IQR rule
- How to identify the relative locations of the mean and median of a distribution form a density curve

Students will be able to:

- Classify variables as categorical or quantitative
- Make and interpret bar graphs for categorical. Make and interpret bar graphs for categorical data
- Calculate marginal and joint relative frequencies from a two-way table
- Calculate conditional relative frequencies from a two-way table
- Use bar graphs to compare distributions of categorical data
- Describe the nature of the association between two categorical variables
- Make and interpret dotplots, stemplots, and histograms of quantitative data
- Describe the overall pattern (shape, center, and variability) of a distribution and identify any major departures from the pattern (outliers)
- Compare distributions of quantitative data using dotplots, stemplots, and histograms
- Calculate measures of center (mean, median) for a distribution of quantitative data
- Calculate and interpret measures of variability (range, standard deviation, IQR) for a distribution of quantitative data
- Explain how outliers and skewness affect measures of center and variability
- Make and interpret boxplots of quantitative data
- Use boxplots and numerical summaries to compare distributions of quantitative data
- Find and interpret the percentile of an individual value within a distribution of data
- Estimate percentiles and individual values using a cumulative relative frequency graph
- Find and interpret the standardized score (z-score) of an individual value within a distribution of data
- Describe the effect of adding, subtracting, multiplying or dividing by a constant on the shape, center, and variability of a distribution of data
- Use a density curve to model distributions of quantitative data
- Use the 68-95-99.7 rule to estimate the proportion of values in a specified interval, or the value that corresponds to a given percentile in a Normal Distribution
- Find the proportion of values in a specified interval in a Normal Distribution using Table A or technology
- Find the value that corresponds to a given percentile in a Normal Distribution using Table A or technology
- Determine whether a distribution of data is approximately Normal from graphical and numerical evidence

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: Textbook & College Board 2019 Binder

Unit 2: Exploring Two-Variable Data

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

Students will explore relationships in two-variable categorical or quantitative data sets. They will use graphical and numerical methods to investigate an association between two categorical variables. Students will describe form, direction, strength, and unusual features for an association between two quantitative variables. They will assess correlation and, if appropriate, use a linear model to predict values of the response variable from values of the explanatory variable. Students will interpret the least-squares regression line in context, analyze prediction errors (residuals), and explore departures from a linear pattern.

Recommended Pacing

8 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with Mathematics
5	Use appropriate tools strategically
6	Attend to precision

Standard: S-ID.B Summarize, represent, and interpret data on two categorical and quantitative variables

CPI #	Cumulative Progress Indicator (CPI)
5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data
6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. c. Fit a linear function for a scatter plot that suggests a linear association.

Standard: S-ID.C Interpret linear models

CPI #	Cumulative Progress Indicator (CPI)
7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data
8	Compute (using technology) and interpret the correlation coefficient of a linear fit.
9	Distinguish between correlation and causation

New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- We look for relationships among variables to help explain patterns and make predictions.
- Direction, unusual features, form and strength of the association are addressed as important characteristics between two variables in a scatterplot.
- Least-squares Regression Lines are used as models for relationships between variables that have a linear relationship.

Unit Essential Questions

- What is correlation all about?
- What do the characteristics of a scatterplot tell us about data?
- What does correlation doesn't mean causation mean?

Objectives

Students will know:

- Vocabulary: response variable, explanatory variable, scatterplot, positive/negative/no association, correlation, regression line, extrapolation, residual, y-intercept, slope, least-squares regression line, residual plot, standard deviation of the residuals (s), coefficient of determination
- How to identify the equation of the least squares regression line, correlation coefficient, coefficient of determination and s from a computer output

Students will be able to:

- Distinguish between explanatory and response variables for quantitative data
- Make a scatterplot to display the relationship between two quantitative variables
- Describe the direction, form, and strength of a relationship displayed in a scatterplot and identify unusual features
- Interpret the correlation
- Understand the basic properties of correlation, including how the correlation is influenced by outliers
- Distinguish correlation from causation
- Make predictions using regression lines, keeping in mind the danger of extrapolation
- Calculate a residual and interpret a residual
- Interpret the slope and y-intercept of a regression line
- Determine the equation of a least-squares regression line using technology
- Construct and interpret residual plots to assess whether a regression model is appropriate
- Interpret the standard deviation of the residuals and r^2 and use these values to assess how well a least-squares regression line models the relationship between two variables
- Describe how the least-squares regression line, standard deviation of the residuals, and r^2 are influenced by outliers
- Find the slope and the y-intercept of the least-squares regression line from the means and standard deviations of x and y and their correlation

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: Textbook & College Board 2019 Binder

Unit 3: Collecting Data

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

Depending on how data are collected, we may or may not be able to generalize findings or establish evidence of causal relationships. If random selection is not used to obtain a sample from a population, bias may result and statistics from the sample cannot be generalized to the population. For data collected using well-designed experiments, statistically significant differences between or among experimental treatment groups are evidence that the treatments caused by the effect. Students learn important principles of sampling and experiment design.

Recommended Pacing

11 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with Mathematics
5	Use appropriate tools strategically
6	Attend to precision

Standard: S-IC.A Understand and evaluate random processes underlying statistical experiments

CPI #	Cumulative Progress Indicator (CPI)
1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Standard: S-IC.B Make inferences and justify conclusions from sample surveys, experiments, and observational studies

CPI #	Cumulative Progress Indicator (CPI)
3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant
6	Evaluate reports based on data.

New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- Populations are too difficult to collect because of time and money.
- Samples can be biased; There are different ways to make sure we have unbiased samples.
- Observational studies look at relationships and associations between two variables.
- Experiments impose treatments to see if there is a cause and effect relationship.
- Experiments can be considered statistically significant.

Unit Essential Questions

- What is the difference between a population and sample?
- How do we avoid bias?
- What is a stratified random sampling vs a cluster sample?
- What are the differences between an observational study and an experiment design?
- What is sampling variability?

Objectives

Students will know:

- Vocabulary: Population, census, sample, sample survey, convenience sampling, bias, voluntary response sampling, random sampling, simple random sample, stratified random sample, cluster sampling, undercoverage, nonresponse, response bias, observational study, response variable, explanatory variable, confounding, experiment, placebo, treatment, experimental unit, subjects, factor, levels, control group, placebo effect, double-blind, single-blind, random assignment, control, replication, completely randomized design, randomized block design, block, matched pairs design, sampling variability, statistically significant
- How to identify the population and sample in a statistical study
- How to identify voluntary response sampling and convenience sampling
- How to identify the explanatory and response variables in an observational study and an experiment
- How to identify the experimental units and treatments in an experiment
- How to identify when it is appropriate to make an inference about a population and when it is appropriate to make an inference about cause and effect

Students will be able to:

- Explain how voluntary response sampling and convenience sampling can lead to bias
- Describe how to select a simple random sample with technology or a table of random digits
- Describe how to select a sample using stratified random sampling and cluster sampling, distinguish stratified random sampling from cluster sampling, and give an advantage of each method
- Explain how undercoverage, nonresponse, question wording, and other aspects of a sample survey can lead to bias
- Explain the concept of confounding and how it limits the ability to make cause-and-effect conclusions
- Distinguish between an observational study and an experiment
- Describe the placebo effect and the purpose of blinding in an experiment
- Describe how to randomly assign treatments in an experiment using slips of paper, technology, or a table of random digits
- Explain the purpose of comparison, random assignment, control, and replication in an experiment
- Describe a completely randomized design for an experiment
- Describe a randomized block design and a matched pairs design for an experiment and explain the purpose of blocking in an experiment
- Explain the concept of sampling variability when making an inference about a population and how sample size affects sampling variability
- Explain the meaning of statistically significant in the context of an experiment and use simulation to determine if the results of an experiment are statistically significant
- Evaluate if a statistical study has been carried out in an ethical manner

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics

Suggested Resources: Textbook & College Board 2019 Binder

Unit 4: Probability, Random Variables, and Probability Distributions

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

Probabilistic reasoning allows statisticians to quantify the likelihood of random events over the long run and to make statistical inferences. Simulations and concrete examples can help students to understand the abstract definitions and calculations of probability. Students will build on understandings of simulated or empirical data distributions and fundamental principles of probability to represent, interpret, and calculate parameters for theoretical probability distributions for discrete random variables/ Interpretations of probabilities and parameters associated with a probability distribution should use appropriate units and relate to the context of the situation.

Recommended Pacing

17 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct a viable argument and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically

Standard: S-CP.A Understand independence and conditional probability and use them to interpret data

CPI #	Cumulative Progress Indicator (CPI)
1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Standard: S-CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model	
CPI #	**Cumulative Progress Indicator (CPI)**
6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
Standard: S-MD.A Calculate expected values and use them to solve problems	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?
Standard: S-MD.B Use probability to evaluate outcomes of decisions	
CPI #	**Cumulative Progress Indicator (CPI)**
5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.

New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- Simulations can be used to estimate probabilities and make decisions.
- Chance behavior can be described by a probability model.
- There is a connection between conditional probability and conditional relative frequencies.
- There is a distinct difference between a discrete and a continuous random variable.
- Linear transformations have the same effect on the probability distribution of a random variable as they have on a distribution of data.
- The Binomial and Geometric random variables are two common types of discrete random variables.

Unit Essential Questions

- How does chance show up in our daily lives?
- What are useful ways to organize the information provided in a conditional probability?
- How are the general multiplication rule, the multiplication rule for independent events, and the conditional formula related?
- What numerical summaries change when each value in a distribution is multiplied and/or added by a number? Why do the changes happen?
- How does a distribution change when two or more random variables are combined (sum or difference)?
- What are the conditions of a binomial and geometric distribution?

Objectives

Students will know:

- Vocabulary: probability, law of large numbers, simulation, probability model, sample space, event, complement rule, mutually exclusive, addition rule, Venn diagram, intersection/union, conditioning probability, independent events, multiplication rule, tree diagram, random variable, probability distribution, discrete random variable, expected value, continuous random variable, independent random variables, binomial setting, binomial distribution, binomial coefficient, 10% condition, large counts condition, geometric setting, geometric distribution

Students will be able to:

- Interpret probability as a long-run relative frequency
- Use simulation to model chance behavior
- Give a probability model for a chance process with equally likely outcomes and use it to find the probability of an event
- Use basic probability rules, including the complement rule and the addition rule for mutually exclusive events
- Use a two-way table or Venn diagram to model a chance process and calculate probabilities involving two events

- Apply the general addition rule to calculate probabilities
- Calculate and interpret conditional probabilities
- Determine if two events are independent
- Use the general multiplication rule to calculate probabilities
- Use a tree diagram to model a chance process involving a sequence of outcomes and to calculate probabilities
- When appropriate, use the multiplication rule for independent events to calculate probabilities
- Use the probability distribution of a discrete random variable to calculate the probability of an event
- Make a histogram to display the probability distribution of a discrete random variable and describe its shape
- Calculate and interpret the mean (expected value) of a discrete random variable
- Calculate and interpret the standard deviation of a discrete random variable
- Use the probability distribution of a continuous random variable (uniform or Normal) to calculate the probability of an event
- Describe the effect of adding or subtracting a constant or multiplying or dividing by a constant on the probability distribution of a random variable
- Calculate the mean and standard deviation of the sum or difference of random variables
- Find probabilities involving the sum or difference of independent Normal random variables
- Determine whether the conditions for a binomial setting are met
- Calculate and interpret probabilities involving binomial distributions
- Calculate the mean and standard deviation of a binomial random variable. Interpret these values
- When appropriate, use the Normal approximation to the binomial distribution to calculate probabilities
- Find probabilities involving geometric random variables

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: Textbook & College Board 2019 Binder

Unit 5: Sampling Distributions

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

This unit applies probabilistic reasoning to sampling, introducing students to sampling distributions of statistics they will use when performing inference. Students should understand that sample statistics can be used to estimate the corresponding population parameters and that measures of center (mean) and variability (standard deviation) for these sampling distributions can be determined directly from the population parameters when certain sampling criteria are met. For large enough samples from any population, these sampling distributions can be approximated by a normal distribution. Simulating sampling distributions helps students to understand how the values of statistics vary in repeated random sampling from populations with known parameters.

Recommended Pacing

7 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision

Standard: S-ID.A

CPI #	Cumulative Progress Indicator (CPI)
2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Technical Reading

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

New Jersey Student Learning Standards for 21st Century Life and Careers

Career Ready Practices

CPI #	Cumulative Progress Indicator (CPI)
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.

New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
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Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- There is a crucial distinction between a parameter and a statistic.
- Statistics vary.
- The sampling distribution of proportions and means approach a Normal distribution as *n* increases.

Unit Essential Questions

- What is the difference between a parameter and a statistic?
- How do we distinguish between using the binomial formula and the normal approximation when investigating a proportion?
- What is the difference between the population distribution, the distribution of the sample, and the sampling distribution of a sample statistic?

Objectives

Students will know:

- Vocabulary: statistic, parameter, sampling variability, sampling distribution, unbiased estimator, sampling distribution of a sample proportion, sampling distribution of a sample mean, central limit theorem

Students will be able to:

- Distinguish between a parameter and a statistic
- Create a sampling distribution using all possible samples from a small population
- Use the sampling distribution of a statistic to evaluate a claim about a parameter
- Distinguish among the distribution of a population, the distribution of a sample, and the sampling distribution of a statistic
- Determine if a statistic is an unbiased estimator of a population parameter
- Describe the relationship between sample size and the variability of a statistic
- Calculate the mean and standard deviation of the sampling distribution of a sample proportion $\hat{p}$ and interpret the standard deviation
- Determine if the sampling distribution of $\hat{p}$ is approximately Normal
- If appropriate, use a Normal distribution to calculate probabilities involving $\hat{p}$
- Calculate the mean and standard deviation of the sampling distribution of a sample mean $\underline{x}$ and interpret the standard deviation
- Explain how the shape of the sampling distribution of $\underline{x}$ is affected by the shape of the population distribution and the sample size
- If appropriate, use a Normal distribution to calculate probabilities involving $\underline{x}$

Evidence of Learning
Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Resources
Core Text: The Practice of Statistics **Suggested Resources: Textbook & College Board 2019 Binder**

Unit 6: Inference for Categorical Data: Proportions

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

This unit introduces statistical inference, which will continue through the end of the course. Students will analyze categorical data to make inferences about binomial population proportions. Provided conditions are met, students will use statistical inference to construct and interpret confidence intervals to estimate population proportions and perform significance tests to evaluate claims about population proportions. Students begin by learning inference procedures for one proportion and then examine inference methods for a difference between two proportions. They will also interpret the two types of errors that can be made in a significance test, their probabilities, and the possible consequences in context.

Recommended Pacing

12 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Technical Reading

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

New Jersey Student Learning Standards for 21st Century Life and Careers

Career Ready Practices

CPI #	Cumulative Progress Indicator (CPI)
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.

New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- There is a difference between why we use a confidence interval compared to a hypothesis test for proportions.
- Understand how the difference between two sample proportions varies due to the chance involved in random selection is essential for drawing conclusions based on observed difference in proportions in a study.

Unit Essential Questions

- What are the conditions for using inference for population proportions?
- How does the formula change for the standard deviation for proportions in the confidence interval and the hypothesis test?

Objectives

Students will know:

- Vocabulary: point estimator, point estimate, confidence interval, confidence level, margin of error, critical value, standard error, significance test, null hypothesis, alternative hypothesis, one-sided and two-sided alternative hypotheses, p-value, significance level, type I error, type II error, standardized test statistic, power
- How to identify an appropriate point estimator for a proportion

Students will be able to:

- Calculate the value of a point estimate
- Interpret a confidence interval in context
- Determine the point estimate and margin of error from a confidence interval
- Use a confidence interval to make a decision about the value of a parameter
- Interpret a confidence level in context
- Describe how the sample size and confidence level affect the margin of error
- Explain how practical issue like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval
- State and check the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion
- Determine the critical value for calculating a C% confidence interval for a population proportion using a table or technology
- Construct and interpret a confidence interval for a population proportion
- Determine the sample size required to obtain a C% confidence interval for a population proportion with a specified margin of error
- State appropriate hypotheses for a significance test about a population parameter
- Interpret a p-value in context
- Make an appropriate conclusion for a significance test

- Interpret Type I and Type II error in context. Give a consequence of each error in a given setting
- State and check the Random, 10%, and Large Counts conditions for performing a significance test about a population proportion
- Calculate the standardized test statistic and P-value for a test about a population proportion
- Perform a significance test about a population proportion
- Use a confidence interval to make a conclusion for a two-sided test about a population proportion
- Interpret the power of a significance test and describe what factors affect the power of a test
- Describe the shape, center, and variability of the sampling distribution of $\hat{p}_1 - \hat{p}_2$
- Determine whether the conditions are met for doing inference about a difference between two proportions
- Construct and interpret a confidence interval for a difference between two proportions
- Calculate the standardized test statistic and P-value for a test about a difference between two proportions
- Perform a significance test about a difference between two proportions

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: Textbook & College Board 2019 Binder

Unit 7: Inference for Quantitative Data: Means

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

In this unit, students will analyze quantitative data to make inferences about population means. Students should understand that t* and t-tests are used for inference with means when the population standard deviation, σ, is not known. Using s for σ in the formula for z gives a slightly different value, t, whose distribution, which depends on sample size, has more area in the tails than a normal distribution. The boundaries for rejecting a null hypothesis using a t-distribution tend to be further from the mean than a normal distribution. Students should understand how and why conditions for inference with proportions and means are similar and different.

Recommended Pacing

13 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision

Instructional Focus

Unit Enduring Understandings

- There is a difference between why we use a confidence interval compared to a hypothesis test for means.
- There is a difference between a two sample inference for means verse a paired data test for means.
- The Central Limit theorem is important in understanding what the sampling distribution of the sample mean approaches as n increases.

Unit Essential Questions

- When do we use a t distribution instead of a z distribution to analyze means?
- How does a t distribution compare to a z distribution?
- Why would we choose to do a paired test for means?

Objectives

Students will know:

- Vocabulary: t distribution, paired data
- How to identify an appropriate point estimator for a mean

Students will be able to:

- Determine the critical value for calculating a C% confidence interval for a population mean using a table or technology
- State and check the Random, 10%, and Normal/Large sample conditions for constructing a confidence interval for a population mean
- Construct and interpret a confidence interval for a population mean
- Determine the sample size required to obtain a C% conficent interval for a population mean with a specified margin of error

- State and check the Random, 10%, and Normal-Large Sample conditions for performing a significance test about a population mean
- Calculate the standardized test statistic and P-value for a test about a population mean
- Perform a significance test about a population mean
- Use a confidence interval to make a conclusion for a two-sided test about a population mean
- Describe the shape, center, and variability of the sampling distribution of $\underline{x_1} - \underline{x_2}$
- Determine whether the conditions are met for doing inference about a difference between two means
- Construct and interpret a confidence interval for a difference between two means
- Calculate the standardized test statistic and P-value for a test about a difference between two means
- Perform a significance test about a difference between two means
- Analyze the distribution of differences in a paired data set using graphs and summary statistics
- Construct and interpret a confidence interval for a mean difference
- Perform a significance test about a mean difference
- Determine when it is appropriate to use paired t procedures versus two-sample t procedures

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: Textbook & College Board 2019 Binder

Unit 8: Inference for Categorical Data: Chi-Square

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

Chi-square tests are used when there are two or more categories. Students need to know how to select from the following tests: the chi-square goodness of fit test (for a distribution of proportions of one categorical variables within a single population), the chi-square test for independence o(for associations between categorical variables within a single population), or the chi-square test for homogeneity (for comparing distributions of a categorical variable across populations or treatments). To integrate conceptual understanding teacher can make connections between frequency tables, conditional probability, and calculating expected counts. The chi-square statistic is introduced to measure the distance between the observed and expected counts relative to expected counts.

Recommended Pacing

6 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Technical Reading

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

New Jersey Student Learning Standards for 21st Century Life and Careers

Career Ready Practices

CPI #	Cumulative Progress Indicator (CPI)
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.

New Jersey Student Learning Standards for Technology

CPI #	Cumulative Progress Indicator (CPI)
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

Interdisciplinary Standards Science

CPI #	Science and Engineering Practices
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- Chi-squared tests always compare the observed distribution of a categorical variable with a hypothesized distribution.
- Chi-squared test for goodness of fit tests one-variable, and chi-squared test for homogeneity and chi-squared test for independence test two-variables.

Unit Essential Questions

- What is the shape of a Chi-Squared distribution?
- What are the differences between a chi-squared test for homogeneity and chi-squared test for independence?

Objectives

Students will know:

- Vocabulary: chi-square statistic, chi-square distribution

Students will be able to:

- State appropriate hypotheses and compute the expected counts and chi-square test statistic for a chi-square test for goodness of fit
- State and check the Random, 10%, and Large Counts conditions for performing a chi-square test for goodness of fit
- Calculate the degrees of freedom and P-value for a chi-square test for goodness of fit
- Perform a chi-square test for goodness of fit
- Conduct a follow-up analysis when the results of a chi-square test are statistically significant
- State appropriate hypotheses and compute the expected counts and chi-square test statistic for a chi-square test based on data in a two-way table
- State and check the Random, 10%, and Large Counts conditions for a chi-squared test based on data in a two-way table
- Calculate the degrees of freedom and P-value for a chi-square test based on data in a two-way table
- Perform a chi-square for homogeneity
- Perform a chi-squared test for independence
- Choose the appropriate chi-square test in a given setting

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: Textbook & College Board 2019 Binder

Unit 9: Inference for Quantitative Data: Slopes

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

Students may be surprised to learn there is variability in slope. In this unit, students will learn how to construct confidence intervals for and perform significance tests about the slope of a population regression line for a particular set of bivariate quantitative data.

Recommended Pacing

7 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Technical Reading

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

New Jersey Student Learning Standards for 21st Century Life and Careers

Career Ready Practices

CPI #	Cumulative Progress Indicator (CPI)
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.

New Jersey Student Learning Standards for Technology

CPI #	Cumulative Progress Indicator (CPI)
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- We have to use inference to create an interval estimate for, or to make decisions about, the true slope of a regression line.
- Computer output gives the necessary information to find the equation of the LSRL, relevant statistics for the linear fit, as well as the information for inference.
- Many pairs of quantitative variables have nonlinear relationships.

Unit Essential Questions

- How is inference in a regression setting similar to proportions and means?
- How do we transform nonlinear data to and find a linear model that is appropriate?

Objectives

Students will know:

- Vocabulary: population regression line, sample regression line
- How to identify the value of b_o, b_1, s, and SE_{b_1} from a computer output

Students will be able to:

- Check the conditions for performing inference about the slope β_1 of the population (true) regression line
- Interpret the values of b_o, b_1, s, and SE_{b_1} in context
- Construct and interpret a confidence interval for the slope β_1 of the population (true) regression line
- Perform a significance test about the slope β_1 of the population (true) regression line
- Use transformations involving powers and roots to find a power model that describes the relationship between two quantitative variables, and use the model to make predictions
- Use transformations involving logarithms to find a power model that describes the relationship between two quantitative variables, and use the model to make predictions
- Use transformations involving logarithms to find an exponential model that describes the relationship between two quantitative variables, and use the model to make predictions
- Determine which of several transformations does a better job of producing a linear relationship

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: Textbook & College Board 2019 Binder

Unit 10: Project

Content Area: Mathematics

Course & Grade Level: AP Statistics, 10-12

Summary and Rationale

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. In this unit, the students will have the opportunity to use statistics to model a real-life study with data. Students will form a hypothesis, collect data and analyze the data.

Recommended Pacing

29 Days

New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Technical Reading

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

New Jersey Student Learning Standards for 21st Century Life and Careers

Career Ready Practices

CPI #	Cumulative Progress Indicator (CPI)
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP7.	Employ valid and reliable research strategies.

New Jersey Student Learning Standards for Technology

CPI #	Cumulative Progress Indicator (CPI)
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

Interdisciplinary Standards Science	
CPI #	**Science and Engineering Practices**
HS-PS1-4	Develop a model based on evidence to illustrate relationships between systems or between components of a system.

Instructional Focus

Unit Enduring Understandings

- We are responsible to generate knowledge to share
- Statistics quantify our everyday experience

Unit Essential Questions

- How does data and statistics apply to your everyday world?
- What information can you quantify for the greater good?

Objectives

Students will know:

- Vocabulary associated with the course
- How to be a critical consumer of statistics, intelligently questioning and analyzing uses and abuses of statistics in the world outside the classroom
- Techniques to explore sets of data, identifying patterns and departures from patterns, and determining the significance of these departures
- How to decide what and how to measure when planning a study
- How to apply the concepts of statistical inference to confirm statistical models
- How to apply the techniques they have studied in projects of their own choosing, reflecting their individual interests and strengths

Students will be able to:

- Work in small groups to research topics of personal interest
- Select one topic and design either a sample study or an experiment
- Carry out their sample study or experiment
- Carry out the appropriate inferential statistics procedures on their data
- Interpret and communicate their findings, both in a written report and a class presentation
- Use graphing calculators, computers and the Internet for research and data analysis

Evidence of Learning

Assessment Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: The Practice of Statistics
Suggested Resources: