West Windsor-Plainsboro Regional School District
Pre-Calculus Curriculum

Unit 1: Foundations of Algebra, Functions & Relations

Content Area: Mathematics

Course & Grade Level: 11-12

Summary and Rationale

Pre-calculus is a continuation of the more advanced concepts of algebra and geometry integrated with the study of analytic and triangle trigonometry; it is a segue to calculus from algebra. This unit will focus on reinforcing and extending upon algebraic concepts developed in prerequisite courses, which will be utilized throughout the pre-calculus curriculum. These concepts will include factoring, quadratic formula, graphing and analyzing functions, solving equations and inequalities, and more. These ideas are crucial to the development of higher-level mathematical reasoning incorporated throughout the pre-calculus course.

Recommended Pacing

10 Days

New Jersey Student Learning Standards for Mathematics

Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them.
3	Construct viable arguments and critique the reasoning of others.
7	Look for and make use of structure.

Standard: Interpreting Functions F-IF.A: Understand the concept of a function and use function notation

CPI #	Cumulative Progress Indicator (CPI)
2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

Standard: Interpreting Functions F-IF.C: Analyze functions using different representations

CPI #	Cumulative Progress Indicator (CPI)
7b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Standard: Interpreting Functions F-BF.B: Build new functions from existing functions

CPI #	Cumulative Progress Indicator (CPI)
3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9-10 texts and topics*.
Standard: Science Integration of Knowledge and Ideas	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11	Use technology to enhance productivity.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Instructional Focus

Unit Enduring Understandings

- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.
- Mathematical models can be used to describe and quantify physical relationships.
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra.

Unit Essential Questions

- How can you describe the behavior and characteristics of a graph of a function?
- How do algebraic concepts help us make sense of phenomena and solve real-life problems?
- How can we use physical models to clarify mathematical relationships?

Objectives

Students will know:

- Quadratic formula
- Factoring through a variety of methods
- Solving algebraic equations and inequalities
- Graphing and analyzing properties of functions

Students will be able to:

- Approach problem-solving by focusing on understanding concepts rather than rote use of procedures and formulas.
- Verify the correctness of their solutions through a variety of methods.
- Graph relations and be able to determine whether or not the relation is a function.
- Identify different types of functions.
- Apply transformations to graphs of functions.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Precalculus, Miller & Gerken (2017)

Suggested Resources:

Unit 2: Conics

Content Area: Mathematics

Course & Grade Level: 11-12

Summary and Rationale

The study of conic sections is a combination of algebra and geometry. Defined as the intersection of a double napped right circular cone and a plane, each type of conic section can be represented as an equation in the Cartesian, and later, the polar plane. These conics (circles, ellipses, parabolas, and hyperbolas) are applied in real-world situations, such as relative distance from an epicenter, orbits of planets, flashlights, and satellites.

Recommended Pacing

10 Days

New Jersey Student Learning Standards for Mathematics

Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
6	Attend to precision.
7	Look for and make use of structure.

Standard: Expressing Geometric Properties with Equations G-GPE.A:
Translate between the geometric description and the equation for a conic section

CPI #	Cumulative Progress Indicator (CPI)
1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2	Derive the equation of a parabola given a focus and directrix.
3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Science Key Ideas and Details

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9-10 texts and topics*.
Standard: Science Integration of Knowledge and Ideas	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11	Use technology to enhance productivity.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Instructional Focus

Unit Enduring Understandings

- The various members of the families of functions have similarities and differences among them.
- The differences between Euclidean and Cartesian approaches to geometry can provide distinct insight with respect to problem solving.
- Investigation and exploration are essential to the development of mathematical ideas.
- Algebraic, graphical, and numerical representations can be used to generalize patterns and relationships.

Unit Essential Questions

- What do the key components (i.e. respective axes, foci, vertices, center, directrix, eccentricity) of the graph of a conic tell us?
- Which method is more efficient to solve a problem? Geometrically or algebraically?
- How do conic sections model real-world phenomena?

Objectives

Students will know:

- Terms: latus chord, directrix, center, focus, eccentricity, definition of a conic, standard form, vertex, major axis, minor axis, transverse axis, asymptotes.
- Equations: General Form of Conic, Standard Form of Circle, Ellipse, Parabola, and Hyperbola.

Students will be able to:

- Compare and contrast the equations of parabolas, circles, ellipses and hyperbolas.
- Graph the equations of parabolas, circles, ellipses and hyperbolas.
- Solve applications involving parabolas, circles, ellipses and hyperbolas.
- Identify a conic from its general form.
- Determine which conic section from the Cartesian equation and subsequently complete the square to rewrite the equation in standard form and sketch.
- Determine the equation of specific conics given particular characteristics; i.e. foci, equations of asymptotes, vertices, etc.
- Determine the eccentricity of a conic section and how it affects the shape of the graph.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Precalculus, Miller & Gerken (2017)

Suggested Resources:

Unit 3: Trigonometry

Content Area: Mathematics

Course & Grade Level: 11-12

Summary and Rationale

Trigonometric functions are essential in modeling periodic and oscillating behavior. This unit is designed to provide three approaches: Euclidean, Cartesian, and Polar, which are used in developing both a graphical and algebraic sense of trigonometric functions. Solving real world situations in a geometrical context starts by establishing a foundation in the unit circle, which subsequently leads to the development of writing trigonometric functions to model these situations.

After developing a knowledgebase for trigonometry, an in-depth algebraic and graphical study of trigonometric functions is examined in this unit. A major component of analytic trigonometry is solving and graphing equations involving trigonometric expressions. Algebraic relationships, such as inverses and identities, are explored for the purpose of solving these equations.

The polar coordinate system builds upon the idea of the unit circle. The concept of establishing a point using the distance from the origin (pole) and an angle in standard position is vastly different than establishing a point using horizontal and vertical components. The polar coordinate system is appreciated through its connection with the rectangular coordinate system. This allows for the development of conversions to move between each system.

Recommended Pacing

72 Days

New Jersey Student Learning Standards for Mathematics

Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
6	Attend to precision.
7	Look for and make use of structure.

Standard: Trigonometric Functions F-TF.A: Extend the domain of trigonometric functions using the unit circle

CPI#	Cumulative Progress Indicator
1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle
2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle

3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number
4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Standard: Trigonometric Functions F-TF.B: Model periodic phenomena with trigonometric functions	
CPI#	**Cumulative Progress Indicator**
5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline
6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context
Standard: Trigonometric Functions F-TF.C: Prove and apply trigonometric identities	
CPI#	**Cumulative Progress Indicator (CPI)**
8	Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Standard: Similarity, Right Triangles, and Trigonometry G-SRT.D: Apply trigonometry to general triangles	
CPI #	**Cumulative Progress Indicator (CPI)**
9	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4.	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9-10 texts and topics*.

Standard: Science Integration of Knowledge and Ideas	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7.	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11	Use technology to enhance productivity.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
New Jersey Student Learning Standards for Science	
CPI #	**Cumulative Progress Indicator (CPI)**
HS-PS4-1	Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

Instructional Focus

Unit Enduring Understandings

- Patterns, relations, and functions can be used as tools to best describe and help explain real-life situations.
- Oscillating functions can be used to describe real-life phenomena.
- As problems become more complex, there are many ways to solve them. Mathematicians look for the most efficient method.
- There are alternatives to the Cartesian coordinate system, which may work better to describe some real-world phenomena.

Unit Essential Questions

- How can we use mathematical models to describe and clarify physical and mathematical relationships?
- What do the key components (i.e. amplitude, period, phase shift, vertical shift) of the graph of a trigonometric function represent?
- How does the study of trigonometry and polar graphs relate to real-world phenomena?
- How are the laws of sines and cosines applied in the real world for example navigation and surveying applications?
- When is it more appropriate to use a Euclidean or a Cartesian approach to trigonometry?
- How are the rectangular and polar coordinate system related, algebraically and graphically?

Objectives

Students will know:

- Terms: Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Radian, Degree, Sinusoidal, Unit Circle, Periodic Functions, Amplitude, Phase Shift, Period, Identities, Even and Odd Functions, Harmonic Motion, Pole, Polar axis, Polar coordinates, Limacon, Rose curves, Cardioid, Polar form.
- Formulas: Arc length, Area of a Sector, Laws of Sines and Cosines, Heron's Formula, Trigonometric Identities, □ = □□□□□, □ = □□□□□.
- The Six Trigonometric Functions: definitions, properties, graphs, inverses (of sine, cosine, tangent)
- The sum and difference formulas
- The double angle formulas

Students will be able to:

- Determine the domain, range, zeros, amplitude, phase shift and period of sinusoidal functions.
- Graph sinusoidal functions and their transformations without a graphing calculator.
- Use even-odd properties to find the exact values of the trigonometric functions.
- Solve real-world problems applying trigonometric functions and model periodic behavior.
- Simplify trigonometric expressions.
- Find arc length and area of a sector of a circle.
- Prove trigonometric identities.
- Apply identities, graphs and/or technology to solve more difficult trigonometric equations.
- Solve and apply problems using right triangle trigonometry.
- Use Law of Sines and Law of Cosines to solve triangles and real-world problems.
- Determine when it is appropriate to use Law of Sines (AAS, ASA, SSA Triangles) and Law of Cosines (SAS, SSS Triangles).
- Derive the formula for the area of a triangle in the SAS case (using the sine function) and apply it to problem solving.
- Apply the sum and difference formulas, and derive and apply the double angle formula.
- Convert points and equations from Polar to Cartesian and vice versa.
- Graph polar equations (lines, circles, limacons, and rose curves) and write polar equations from graphs.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Precalculus, Miller & Gerken (2017)

Suggested Resources:

Unit 4: Systems of Equations & Inequalities

Content Area: Mathematics

Course & Grade Level: 11-12

Summary and Rationale

There are numerous ways of solving linear and nonlinear systems of equations, which will then be extended to systems of inequalities. The purpose of this unit is to provide algebraic, graphical and numerical approaches to solve a variety of systems. Once students are familiar with these methods, they will then apply these approaches to real world phenomena, which includes linear programming. While using these processes, the concept of matrices is utilized as an alternative approach.

Recommended Pacing

15 Days

New Jersey Student Learning Standards for Mathematics

Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them.
3	Construct viable arguments and critique the reasoning of others.
7	Look for and make use of structure.

Standard: Reasoning with Equations and Inequalities A -REI.C: Solve systems of equations

CPI #	Cumulative Progress Indicator (CPI)
7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.
8	Represent a system of linear equations as a single matrix equation in a vector variable.
9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Standard: Reasoning with Equations and Inequalities A -REI.D: Represent and solve equations and inequalities graphically

CPI #	Cumulative Progress Indicator (CPI)
12	Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Science Key Ideas and Details

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4.	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9-10 texts and topics*.
Standard: Science Integration of Knowledge and Ideas	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7.	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11	Use technology to enhance productivity.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Instructional Focus

Unit Enduring Understandings

- Systems can yield no solution, one solution, or multiple solutions.
- Linear programming is used to optimize objective functions restricted by real world parameters.
- Matrices are an effective method for solving systems of equations.
- The solution set to an inequality is the set of all ordered pairs that satisfy the inequality.
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.

Unit Essential Questions

- What is the most efficient way of solving systems of equations and inequalities?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?
- How can linear programming be used to optimize objective functions?

Objectives

Students will know:

- Terms: Systems of linear and non-linear equations and inequalities, linear programming, matrices, determinant, identity matrix, inverse matrix, solution set, elimination, substitution, constraints, restrictions, objective function, vertices, feasible region.

Students will be able to:

- Solve systems of two and three variable linear equations using a variety of methods.

- Solve and graph linear inequalities.
- Add, subtract, and multiply matrices.
- Calculate the determinant of a matrix.
- Determine the inverse of a matrix.
- Model real world situations with systems of equations and inequalities and use them to make predictions.
- Find a pair of values that either maximizes or minimizes a third, dependent variable when given information about the permissible values of two independent variables.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Precalculus, Miller & Gerken (2017)
Suggested Resources:

Unit 5: Polynomial & Rational Functions with Limits

Content Area: Mathematics

Course & Grade Level: 11-12

Summary and Rationale

This unit will focus on the study of polynomial and rational functions. Polynomial functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. This understanding can provide the foundation to make decisions and reasonable predictions for future outcomes of the function.

The algebraic study of rational functions enables mathematicians to symbolize and generalize the rules of arithmetic. This builds a thorough understanding of our number system and the connectedness of the discipline of mathematics. Students will look at the asymptotes of rational functions and use their understanding of the graph to interpret solutions to rational equations.

The concept of limits is essential for developing the underlying theorems used throughout calculus. The mastery of determining one-sided and two-sided limits analytically, graphically, and numerically sets a foundation for differential and integral calculus. The exploration of continuity provides a deeper understanding of how functions work, in addition to special limits, such as infinite limits and limits at infinity.

Recommended Pacing

13 Days

New Jersey Student Learning Standards for Mathematics

Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them.
3	Construct viable arguments and critique the reasoning of others.
7	Look for and make use of structure.

Standard: Arithmetic with Polynomials and Rational Expressions A -APR.B: Understand the relationship between zeros and factors of polynomials

CPI #	Cumulative Progress Indicator (CPI)
2	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Standard: Arithmetic with Polynomials and Rational Expressions A -APR.D: Rewrite Rational Expressions

CPI #	Cumulative Progress Indicator (CPI)
6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Standard: Interpreting Functions F-IF.C: Analyze Functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7d	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4.	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9-10 texts and topics*.
Standard: Science Integration of Knowledge and Ideas	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7.	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for 21st Century Life and Careers	
Career Ready Practices	
CPI #	**Cumulative Progress Indicator (CPI)**
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11	Use technology to enhance productivity.
New Jersey Student Learning Standards for Technology	
CPI #	**Cumulative Progress Indicator (CPI)**
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Instructional Focus

Unit Enduring Understandings

- Polynomial and rational equations are used to model, understand and explain real-life situations.
- Multiple representations of a function are used to identify key information about its graph.
- A limit determines the behavior of a graph as the independent variable approaches a certain value.

Unit Essential Questions

- What does the degree of a polynomial tell you about the function and the relationship it represents?
- How are the factors, zeros, asymptotes, and intercepts related?
- How are limits, asymptotes, and continuity related?
- Why can't the denominator of a rational function be zero and what does it mean if it does?
- What are the advantages and disadvantages of various, equivalent forms of rational expressions?

Objectives

Students will know:

- Terms: Factoring Polynomials, Rational Functions, Limits, One-Sided Limits, vertical and horizontal asymptotes, oblique asymptotes, continuous, continuous on an interval.

Students will be able to:

- Determine the most efficient method in solving polynomial and rational equations and inequalities.
- Solve and graph linear and polynomial inequalities.
- Find the real zeros of a polynomial function.
- Utilize polynomial inequalities in problem solving.
- Graph rational functions by finding zeros, asymptotes, y-intercept and exploring end behavior.
- Write an equation of a given rational function graph.
- Perform long division and synthetic division of polynomials.
- Add, subtract, multiply and divide rational expressions.
- Determine domain, range, holes and asymptotes of rational functions.
- Solve rational equations.
- Relate rational function graphs to the idea of a limit.
- Find the limit graphically and by analyzing a table of values.
- Find one-sided limits in relationship to rational functions.
- Determine where a function is continuous.

Evidence of Learning			
Assessment			
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.			
Competencies for 21st Century Learners			
	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner
Resources			
Core Text: Precalculus, Miller & Gerken (2017) **Suggested Resources:**			

Unit 6: Exponential & Logarithmic Functions (Optional)

Content Area: Mathematics

Course & Grade Level: 11-12

Summary and Rationale

This unit will focus on the study of exponential and logarithmic functions. Exponential functions are used to describe relationships that have a variable rate of change. The logarithmic function, which is the inverse of an exponential function, is studied in further detail from prerequisite courses. Students will look at both types of functions from an analytical, numerical and graphical approach. These functions are used to model real-world situations including population growth, radioactive decay, compound interest, and more.

Recommended Pacing

5 Days

New Jersey Student Learning Standards for Mathematics

Standards for Mathematical Practice

CPI #	Cumulative Progress Indicator (CPI)
1	Make sense of problems and persevere in solving them.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
6	Attend to precision.
7	Look for and make use of structure.

Standard: Interpreting Functions F-IF.C: Analyze Functions using different representations

CPI #	Cumulative Progress Indicator (CPI)
7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8b	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

Standard: Linear and Exponential Models F-LE.A:
Construct and compare linear and exponential models and solve problems

CPI #	Cumulative Progress Indicator (CPI)
1c	Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

New Jersey Student Learning Standards for English Language Arts Companion Standards

Standard: Science Key Ideas and Details

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

Standard: Science Craft and Structure

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.4.	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9-10 texts and topics*.

Standard: Science Integration of Knowledge and Ideas

CPI #	Cumulative Progress Indicator (CPI)
RST.9-10.7.	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

New Jersey Student Learning Standards for 21st Century Life and Careers

Career Ready Practices

CPI #	Cumulative Progress Indicator (CPI)
CRP2.	Apply appropriate academic and technical skills.
CRP4.	Communicate clearly and effectively and with reason
CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11	Use technology to enhance productivity.

New Jersey Student Learning Standards for Technology

CPI #	Cumulative Progress Indicator (CPI)
8.1	All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Instructional Focus

Unit Enduring Understandings

- Mathematicians use exponential and logarithmic equations to model, interpret and explain real-life phenomena.
- An inverse is created by interchanging the independent and dependent variables of a function and used to explain the relationship between exponential and logarithmic functions.

Unit Essential Questions

- How do we model quantities that change over time by the same percentage?
- What is a logarithm and how do mathematicians use them?

Objectives

Students will know:

- Terms: exponential and logarithmic functions, Euler's number e, base, power, exponent, inverse.
- Properties of exponents and logarithms

Students will be able to:

- Graph and transform exponential and logarithmic functions.
- Evaluate expressions containing exponents and logarithms.
- Solve exponential equations using various methods.
- Utilize properties of logarithms to rewrite expressions and solve equations.
- Find the inverse equation when given the equation of a function.
- Graph the inverse of a function.
- Determine domain, range, and asymptotes of exponential, logarithmic and inverse functions.
- Model real world situations with exponential and logarithmic functions and use them to make predictions.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

Competencies for 21st Century Learners

	Collaborative Team Member		Effective Communicator
	Globally Aware, Active, & Responsible Student/Citizen		Information Literate Researcher
	Innovative & Practical Problem Solver		Self-Directed Learner

Resources

Core Text: Precalculus, Miller & Gerken (2017)

Suggested Resources: