West Windsor-Plainsboro Regional School District
AP CALCULUS AB
July 2022

Unit 1: Limits & Continuity	
Content Area: Mathematics	
Course & Grade Level: AP Calculus AB, grades 11 and 12	
Summary and Rationale	
Limits are the foundational concept of calculus. This unit features a multi-representational approach to calculus, with limits, continuity, and their connected theorems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding of how calculus applies limits to develop important ideas, definitions, formulas, and theorems. A sustained emphasis on clear communication of methods, reasoning, justifications, and conclusions is essential for 21st century learners.	
Recommended Pacing	
11 days	
New Jersey Student Learning Standards for	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 10-11 texts and topics.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

New Jersey Student Learning Standards for Computer Science and Design Thinking

CPI #	Cumulative Progress Indicator (CPI)
8.2.12.NT.1	Explain how different groups can contribute to the overall design of a product.

Instructional Focus

Unit Enduring Understandings

- Calculus is the study of the rate of change of values and allows us to find length, area, and volume.
- A limit is the foundation of both differential and integral calculus.
- Continuity is defined by limits and is a prerequisite for many applications in calculus.

Unit Essential Questions

- Why are limits the foundation of calculus and how are they useful?
- How do limits relate to various features of a function and its graph?
- How is continuity used to identify the existence of important values of a function?

Content Understandings

- A limit at a value is defined when the left and right side limits are equal.
- The limit of a function is the value the output of that function approaches as the input approaches some constant.
- A function is continuous when the function value is defined, the limit exists, and the function value equals the limit value.
- The slope of a tangent line at a point is defined as a limit.
- The difference between average rate of change (over a time interval) and instantaneous rate of change (at a single moment).

Content Questions

- How is a limit defined?
- How is continuity defined?
- How is the slope of a tangent line defined?

Objectives

We are learning to/that:

- The informal definition of a limit, definition of continuous.
- Translate among verbal, visual, and algebraic definitions of limits and continuity.
- Evaluate limits using the squeeze theorem.
- Apply the intermediate value theorem.
- Estimate the slope of a line tangent to a given point.
- Estimate the average velocity and instantaneous velocity at a given time.
- Determine the limit of a function by applying the limit laws.
- Evaluate limits approaching infinity.
- Evaluate indeterminate limits, particularly those of difference quotients.
- Apply the definition of continuity.
- Calculate slopes of tangents, velocities, and other rates of change by applying the learned techniques to compute limits.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized tests and NJSLA data.

Resources

Core Text: Calculus for the AP course, 3rd edition, by Sullivan, Miranda, publisher bedford, freeman and worth (bfw), copyright 2020

Unit 2: Differentiation	
Content Area: Mathematics	
Course & Grade Level: AP Calculus AB, grades 11 and 12	
Summary and Rationale	
Differentiation is the mathematical study of change and is a fundamental operation of calculus. This unit features a multi-representational approach to calculus, with limits, derivatives, and their associated theorems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding to describe rates of change of one variable with respect to another or use definite integrals to describe the net change in one variable over an interval of another. This allows students to understand change in a variety of contexts. A sustained emphasis on clear communication of methods, reasoning, justifications, and conclusions is essential for 21st century learners.	
Recommended Pacing	
38 days	
New Jersey Student Learning Standards for	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 10-11 texts and topics.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.NT.1	Explain how different groups can contribute to the overall design of a product.
Interdisciplinary Standards Science	
HS.PS2	Motion and Instability: Forces and Interactions

Science examples: (1) Relate the units of acceleration (m/s2) to the fact that acceleration refers to a change in velocity over time. (2) Reconstruct the units of the universal gravitational constant G by reference to the formula F = Gm1m2/r 2 , instead of having to memorize the units. (2) Attend to units properly when using formulas such as momentum = mass times velocity, etc. (3) Carefully format data displays and graphs, attending to origin, scale, units, and other essential items. NGSS Appendix L, pg. 28

Instructional Focus

Unit Enduring Understandings

- Calculus is the study of the rate of change of values and allows us to find length, area, and volume.
- A derivative is the instantaneous rate of change of a function and can be used in many real-world applications across different fields.
- Derivatives can be used to identify key features of a function and its graph.
- Derivative concepts help explain phenomena in the physical world by examining them in multiple representations.

Unit Essential Questions

- How is an instantaneous rate of change useful?
- How can a derivative be interpreted?
- How can we use derivatives to understand the behavior of functions?

Content Understandings

- A derivative is a function that represents the instantaneous rate of change of another function.
- Find a derivative of multiple types of functions (polynomial, rational, trigonometric, radical, exponential, logarithmic, and inverse).
- Use differentiation techniques (product, quotient, and chain rules) in evaluating the derivative of compositions of functions.
- Find and interpret higher order derivatives for analysis and curve sketching (including max and min).
- When to apply and interpret the Mean Value Theorem.
- Using derivatives to identify and interpret key features of a graph or function.
- Solve applications of derivatives, including related rate and optimization problems.
- Find the antiderivative of a function and differentials.

Content Questions

- What is a derivative and how do we find one?
- What does the Mean Value Theorem tell us, and when do we apply it?
- How do we interpret and solve applications of differentiation, specifically related rate and optimization problems?
- How do we find and interpret an antiderivative and a differential?
- What is the difference between average and instantaneous rates of change?

Objectives

We are learning to/that:

- Apply the definition of derivative as a function.
- Interpret the derivative as a rate of change and find the average & instantaneous velocity.
- Apply the power rule, product rule, quotient rule, and chain rule to evaluate derivatives of functions.

- Differentiate trigonometric functions.
- Apply the method of implicit differentiation.
- Evaluate the derivative of an inverse function at a given point.
- Find the derivative of an exponential function.
- Differentiate logarithmic functions.
- Apply the method of logarithmic differentiation.
- Evaluate derivatives of inverse trigonometric functions.
- Evaluate higher order derivatives.
- Solve related rate applications.
- Find the linearization of a function and use it to approximate values.
- Apply derivatives to find the maximum/minimum values of a function.
- Apply derivatives to find the points of inflection and intervals of concavity of a function.
- Apply L'Hospital's rule in evaluating limits.
- Apply the Mean Value Theorem.
- Analyze and make the connections between a function and its derivative.
- Sketch the curve of a function by applying the first and second derivative tests.
- Connect position, velocity, and acceleration using derivatives for 1-dimensional motion.
- Solve real world optimization problems.
- Determine the antiderivative of a function.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized tests and NJSLA data.

Resources

Core Text: Calculus for the AP course, 3rd edition, by Sullivan, Miranda, publisher bedford, freeman and worth (bfw), copyright 2020

Unit 3: Integration	
Content Area: Mathematics	
Course & Grade Level: AP Calculus AB, grades 11 and 12	
Summary and Rationale	
Along with limits and differentiation, integration is a fundamental operation of calculus. This unit features a multi-representational approach to calculus, with integrals and their related theorems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding to solve problems in mathematics and physics involving the area of an arbitrary shape, the length of a curve, and the volume of a solid, among others. A sustained emphasis on clear communication of methods, reasoning, justifications, and conclusions is essential for 21st century learners.	
Recommended Pacing	
29-31 days	
New Jersey Student Learning Standards for	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
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New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.
Standard: Science Craft and Structure	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 10-11 texts and topics.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.NT.1	Explain how different groups can contribute to the overall design of a product.

Instructional Focus

Unit Enduring Understandings

- Calculus is the study of the rate of change of values and allows us to find length, area, and volume.
- An integral can be used in many real-world applications across different fields.
- Integral concepts help explain phenomena in the physical world by examining them in multiple representations.
- Integrals are connected to derivatives through the Fundamental Theorem of Calculus.

Unit Essential Questions

- How are estimation techniques and limits used to develop the idea of an infinite sum?
- How can an integral be interpreted?
- How are derivatives and integrals related?
- How can we use integrals to understand the behavior of functions?

Content Understandings

- A definite integral allows us to find the area under a curve, and can be interpreted as total and net change of a value.
- What the Fundamental Theorem of Calculus is and how it connects derivatives and integrals.
- The difference between definite and indefinite integrals.
- The importance of the constant of integration and what it represents.
- Differential equations and their graphical representations.
- Geometric applications of integrals in finding area and volume.

Content Questions

- Why are rectangles used to approximate the area in the cartesian plane?
- What methods can we use to integrate different types of functions?
- What is the Fundamental Theorem of Calculus?
- How are derivatives and integrals connected?
- How do we use integration to find total and net change?
- How do we use integration to find volume?

Objectives

We are learning to/that:

- Understand the Fundamental Theorem of Calculus, the Total Change Theorem and the term integral.
- Approximate the area under a curve using Riemann sums or Trapezoidal sums.
- Approximate the total distance traveled by a particle moving along a line.
- Apply the definition of definite integral.
- Apply the Fundamental Theorem of Calculus in evaluating definite and indefinite integrals.
- Solve applications involving the Total Change Theorem.
- Integrate using direct antiderivatives.
- Evaluate definite and indefinite integrals by applying the substitution rule.
- Integrate using completing the square and long division.
- Determine the average value of a function.
- Find the area of a region bounded by two curves.
- Find the volume of a solid obtained by a rotation, using the disk method.
- Find the volume of a solid obtained by a rotation, using the washer method.
- Find the volume of a solid with known cross-sections.

- Solve application problems involving area and volume.
- Construct a slope field.
- Sketch a solution curve using a slope field.
- Solve a differential equation by using separation of variables.
- Analyze exponential growth and decay models.
- Match equations and differential equations with slope fields and solution curves.

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized tests and NJSLA data.

Resources

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