

## West Windsor-Plainsboro Regional School District Algebra 2 <br> August 2022

## Unit 1: Algebra 1 Essentials

## Content Area: Mathematics

Course \& Grade Level: Algebra 2; 9-12

## Summary and Rationale

This unit will focus on the study of functions and relations. Function notation is used to describe relationships in terms of a dependent and independent variable. By studying the characteristics of a function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions. Furthermore, a deep understanding of functions is a crucial part of the foundation for higher level mathematics and physical sciences.

Additionally, students will study linear and absolute value equations. Linear functions are used to describe relationships that have a constant rate of change in terms of a dependent and independent variable. The algebraic study of equations helps mathematicians symbolize and generalize the rules of arithmetic. Students apply their understanding of functions to a variety of situations in order to analyze and explain different numerical relationships.

## Recommended Pacing

28 days

## New Jersey Student Learning Standards for Mathematics

Standard: Standards for Math Practice

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning |
| Standard: A-REI.C Solve systems of equations |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of <br> that equation and a multiple of the other produces a system with the same solutions. |
| 6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of <br> linear equations in two variables. |


| Standard: A-REI.D Represent and solve equations and inequalities graphically |  |
| :--- | :--- |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the <br> coordinate plane, often forming a curve (which could be a line). |

Standard: F-IF.A Understand the concept of a function and use function notation

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Understand that a function from one set (called the domain) to another set (called the range) <br> assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an <br> element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ <br> is the graph of the equation $y=f(x)$. |


| 2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context |
| :---: | :---: |
| Standard: F-IF.B Interpret functions that arise in applications in terms of the context |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity |
| 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. |
| Standard: F-IF.C Analyze functions using different representations |  |
| 7a | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <br> a. Graph linear and quadratic functions and show intercepts, maxima, and minima |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| Standard: F-BF.A Build a function that models a relationship between two quantities |  |
| 1 | Write a function that describes a relationship between two quantities. |
|  | New Jersey Student Learning Standards for English Language Arts Companion Standards |
| Standard: Science Key Ideas and Details |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| RST.6-8.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics |
| RST.6-8.7 | Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). |
| New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 9.4.5.CI. 3 | Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a). |
| 9.4.5.CT. 3 | Describe how digital tools and technology may be used to solve problems. |
| 9.4.5.CT. 4 | Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3). |
| New Jersey Student Learning Standards for Technology |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |
| 8.2 | All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment. |
| Interdisciplinary Standards Science |  |
| MS-PS3-1 | Construct and interpret graphical displays of data to identify linear and nonlinear relationships. |
| Instructional Focus |  |
| Unit Enduring Understandings |  |

- Functions model real world situations using various equations, graphs and tables. These models help mathematicians understand and explain real life data.
- Mathematical solutions do not always represent a real or possible solution.
- There are sets of functions, called families, in which each function is a transformation of a specific function called the parent.


## Unit Essential Questions

- How can mathematicians model complex real world situations?
- What is a function and why are they important when describing data?
- When is the mathematical solution to a problem not a viable solution?
- How can we find an optimal solution to a system of complex, real world, linear constraints?
- What does a graph tell you about a function?


## Content Understandings

- Functions that possess a constant rate of change are classified as linear.
- There are special cases when solving equations that result in one solution, no solution, extraneous solutions, infinite solutions or all real number solutions.
- To solve a system of equations, find a set of values that replace the variables in the equations and make each equation true.
- You can solve a system of equations by writing equivalent systems until the value of one variable is clear. Then substitute to find the values of the other variables.
- Functions can be classified based on rate of change.
- Symmetries in graphs are connected to how the opposite of $x$ and/or $y$ affect the given relation.
- The properties of functions and function operations are used to model and analyze real world applications and quantitative relationships.
- Classifying different types of data allows us to better understand and analyze it, and one such form of identification is discrete vs continuous data. Functions can be discrete or continuous.
- New functions can be built from existing functions by applying transformations.


## Content Questions

- How do the graphs of $y=f(x)+k, y=f(x-h), y=f(-x)$ and $y=k f(x)$ [for positive and negative values of $k$ ] compare to the parent function $f$ ?
- What is a reasonable domain for a given function?
- What is function notation and why is it used?
- How are relationships depicted or communicated?
- What is the difference between a function and a relation?
- How can you determine a linear function by looking at a graph, a table of values or an algebraic equation?
- How do you choose which method to use given an algebraic system or word problem?
- What are the characteristics of some of the basic parent functions?
- How can you determine the number of solutions of a linear system?
- What is the most efficient way to graph given a particular function?


## Objectives

## Students will know:

- The association between relations and functions
- The difference between domain and range
- The properties of transformations of functions
- The sequencing of a list of transformations could result in different graphs


## Students will be able to:

- Compare functions that are represented in different forms (i.e. a table, a rule, a verbal description, a graph and a set of ordered pairs)
- Identify the domain and range from a graph and/or an equation using interval notation.
- Describe the transformation (rigid and non-rigid) of a parent function that would yield the graph of a given function.
- Write a transformed function given a list of transformations to a parent function.
- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another and apply this concept to solve real life applications.
- Graph the equation of a line given in standard, slope intercept or point slope form.
- Find the equation of a line given two points or a point and its slope.
- Determine if two lines are parallel or perpendicular line.
- Use linear functions to solve real life applications.
- Use graphing, substitution and elimination to solve systems of equations with two variables.
- Determine when a system has no solutions, one solution or many solutions.
- Use systems of equations to solve real life applications.
- Graph absolute value equations using transformations.
- Analyze and interpret piecewise functions.
- Graph relations and be able to determine whether or not the relation is a function
- Utilize technology to graph relations (Desmos, TI-84, etc.)
- Use $f(x)$ terminology to evaluate functions
- Utilize variables to represent unknown quantities in real world situations
- Identify all values that make an equation a true statement

| Evidence of Learning |  |  |
| :--- | :--- | :--- |
| Assessment |  |  |
| Assessment plan may include teacher designed formative and summative assessments, a district common <br> assessment, analysis of standardized test and NJSLA data. <br> Competencies for 21 ${ }^{\text {st }}$ Century Learners Collaborative Team Member |  | Effective Communicator |
|  | Globally Aware, Active, \& Responsible Student/Citizen |  |
| Information Literate Researcher |  |  |
|  | Innovative \& Practical Problem Solver |  |
| Resources |  |  |
| Core Text: Big Ideas Math, Algebra 2; Larson and Boswell |  |  |

## Unit 2: Quadratics

## Content Area: Mathematics

Course \& Grade Level: Algebra 2, 9-12

## Summary and Rationale

This unit will focus on the study of quadratic functions. Quadratic functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. These functions can be visually represented by a parabola. By studying the domain, range and rate of change of a quadratic function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions.

There are common forms in which a quadratic can be written and each gives information about the graph and behavior of the function. Understanding the relationships between the characteristics of a quadratic and its equation will forge the connections between the method of graphing a quadratic function based on its algebraic form. Solving quadratic equations leads to zeros of a quadratic function which in turn can translate to the x-intercepts of its graph. Solving techniques will be explored with a new emphasis on including the possibility of complex numbers. In this unit, the solutions of quadratics will no longer be restricted to real numbers.

## Recommended Pacing

31 days

## New Jersey Student Learning Standards for Mathematics

Standard: Standards for Math Practice

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning |

Standard: N-CN.A Perform arithmetic operations with complex numbers

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Know there is a complex number i such that $\mathrm{i} 2=-1$, and every complex number has the form a +bi <br> with a and b real. |
| 2 | Use the relation $\mathrm{i} 2=-1$ and the commutative, associative, and distributive properties to add, <br> subtract, and multiply complex numbers |
| 7 | Solve quadratic equations with real coefficients that have complex solutions. |
| Standard: A-CED.A Create equations that describe numbers or relationships |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Create equations and inequalities in one variable and use them to solve problems. Include equations <br> arising from linear and quadratic functions, and simple rational and exponential functions |

Standard: A-REI.B Solve equations and inequalities in one variable

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 4 | Solve quadratic equations in one variable. . |

$\left.\begin{array}{|l|l|}\hline & \begin{array}{l}\text { a. Use the method of completing the square to transform any quadratic equation in x into an } \\ \text { equation of the form (x-p) ^2 = q that has the same solutions. Derive the quadratic formula from } \\ \text { this form. } \\ \text { b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the }\end{array} \\ \text { square, the quadratic formula and factoring, as appropriate to the initial form of the equation. } \\ \text { Recognize when the quadratic formula gives complex solutions and write them as a bi for real } \\ \text { numbers a and b. }\end{array}\right]$

## Instructional Focus

## Unit Enduring Understandings

- Mathematicians use algebraic and graphical representations to generalize patterns and relationships
- Real world situations involving quadratic relationships can be modeled and solved using multiple representations. Each representation has its advantages and disadvantages.


## Unit Essential Questions

- How can we use mathematical language to describe non-linear change?
- How can we use mathematical models to describe physical and scientific relationships?
- When is an estimate an appropriate solution in a real-life situation?


## Content Understandings

- A graphical representation of quadratic data can be used to analyze and extrapolate pertinent information.
- Quadratic equations and functions can be solved using a variety of methods
- Recognize similarities between a quadratic equation and the graph of the corresponding function
- X-intercepts are potential boundaries between negative and positive outputs and therefore are classified as "critical numbers."
- Analyzing data to determine a constant rate of change or second rate of change helps determine if data is linear or quadratic
- The imaginary unit i was invented to allow us to express complex solutions to quadratic equations
- The solution to a system is graphically represented as the intersection of the two functions

Content Questions

- Which is the most efficient method for solving a quadratic equation?
- What are the advantages and disadvantages of expressing a quadratic in various forms?
- When are some forms more appropriate than others?
- How are quadratic functions related to each other?
- How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?
- What do the solutions of a quadratic equation tell you about its graph?
- How can we use quadratic equations to model and interpret real-life situations?


## Objectives

Students will know:

- Characteristics of quadratic functions.
- A parabola is the graphical representation of a quadratic function
- Procedures for solving quadratic equations and functions
- All the forms of a quadratic function: standard, vertex, intercept form.
- How the parameters for each form of the quadratic function affects the attributes of its corresponding graph
- The Quadratic Formula
- How transformations affect the parent function.

Students will be able to:

- Identify characteristics of a quadratic function
- Graph a quadratic given multiple transformations of the parent function
- Determine transformations from a quadratic equation.
- Graph quadratic functions through standard, vertex, and intercept form
- Solve quadratic equations through the methods of factoring, square roots, completing the square and the quadratic formula
- Algebraically convert between quadratic forms
- Determine maximum/minimum of a parabola through various methods
- Find the $x$-intercepts of a quadratic function through various methods
- Choose and use the best method of solution for a quadratic equation
- Determine the discriminant and the nature of the roots of a quadratic equation
- Relate the nature of the solutions to the discriminant and the graph
- Solve a quadratic equation whose solutions are complex numbers
- Simplify complex numbers
- Add, subtract, and multiply complex numbers
- Write a quadratic equation, given particular information about the function.
- Utilize quadratic equations to model real world situations and use them to make predictions
- Use a graphing calculator to determine the quadratic regression equation of sets of data
$\bullet$


## Evidence of Learning

Assessment
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

## Competencies for $\mathbf{2 1}^{\text {st }}$ Century Learners

|  | Collaborative Team Member |  | Effective Communicator |
| :--- | :--- | :--- | :--- |
|  | Globally Aware, Active, \& Responsible Student/Citizen |  | Information Literate Researcher |
|  | Innovative \& Practical Problem Solver |  | Self-Directed Learner |
| Resources |  |  |  |
| Core Text: Big Ideas Math, Algebra 2; Larson and Boswell |  |  |  |

## Summary and Rationale

This unit will focus on the study of polynomial functions. Polynomial functions are used to describe relationships that have a variable rate of change; in terms of a dependent and independent variable. By studying the domain, range and rate of change of a polynomial function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions.

Students will extend previous connections made between quadratic functions and their graphs. The graphs of polynomial functions have attributes (end behavior, x-intercept behavior, turning points) that are determined by key components of the function itself. One of the key components to analyzing a polynomial function's graph is its x-intercepts which are directly related to the zeros (or roots) of the function. In this unit, students will learn that finding a polynomial function's factored form is the most convenient approach to finding these zeros.

## Recommended Pacing

18 days

## New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 |  |


| 1 | Make sense of problems and persevere in solving them. |
| :--- | :--- |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning |
| Standard: A-APR.B Understand the relationship between zeros and factors of polynomials |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 2 | Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on <br> division by $x$ |
| 3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to <br> construct a rough graph of the function defined by the polynomial. |

Standard: F-IF.B Interpret functions that arise in applications in terms of the context

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 4 | For a function that models a relationship between two quantities, interpret key features of graphs <br> and tables in terms of the quantities, and sketch graphs showing key features given a verbal <br> description of the relationship. Key features include: intercepts; intervals where the function is <br> increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end <br> behavior; and periodicity |
| Standard: F-IF.C Analyze functions using different representations |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 7c | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases <br> and using technology for more complicated cases. |


|  | $\begin{array}{l}\text { c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and } \\ \text { showing end behavior }\end{array}$ |
| :--- | :--- |
| New Jersey Student Learning Standards for English Language Arts |  |
| Companion Standards |  |$]$

- Identifying extrema (mins/maxes) and increasing/decreasing sections of a polynomial function's graph help us to describe its distinguishing characteristics.
- Turning points are extrema that separate increasing and decreasing sections of a polynomial function's graph. There are at most $n-1$ turning points for the graph of a polynomial function of degree $n$.


## Content Questions

- What is the difference between degree and exponent?
- Why must rational roots of a polynomial function be in the form of a factor of the constant over a factor of the leading coefficient?
- Why does synthetic division work? When can it be used?
- What is the difference between turning points and inflection points?
- Using what you know about end behavior, can you draw a quintic with an odd number of turning points?
- What are the benefits of synthetically dividing $x-k$ over evaluating $f(k)$ to determine if $k$ is a root?


## Objectives

## Students will know:

- The number of turning points in the graph of a polynomial function will be at most 1 less than its degree
- Polynomial functions with an odd degree have an even number of turning points in its graph; those with an even degree have an odd number of turning points in its graph
- Remainder Theorem
- Factor Theorem
- Fundamental Theorem of Algebra and its corollary that a polynomial function of degree $n$ must have $n$ complex roots (including multiplicity)


## Students will be able to:

- Identify the end behavior of a polynomial function based on its degree and the sign of its leading coefficient
- Identify the multiplicity of a root and use it to determine x-intercept behavior
- Sketch the graph of a polynomial function based on its end behavior and x-intercept behavior
- Identify increasing and decreasing regions in a polynomial function and express what portion(s) of the domain apply to these regions
- Identify relative and absolute extrema in the graph of a polynomial function and find extrema using technology
- Use synthetic division to test for zeros of a polynomial function
- Evaluate a function using synthetic division (Remainder Theorem)
- Factor polynomial functions completely using techniques from quadratics factoring, factoring by grouping, difference of squares
- Write a polynomial function of least degree with rational coefficients when given certain roots (including irrational and/or imaginary)
- Describe transformations of polynomial functions
- Write transformations of polynomial functions
- Use technology to find models for data sets whose plots appear to be polynomial-based


## Evidence of Learning

## Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

## Competencies for $21^{\text {st }}$ Century Learners

|  | Collaborative Team Member |  | Effective Communicator |
| :--- | :--- | :--- | :--- |
|  | Globally Aware, Active, \& Responsible Student/Citizen |  | Information Literate Researcher |
|  | Innovative \& Practical Problem Solver |  | Self-Directed Learner |
| Resources |  |  |  |

Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

## Unit 4: Rational Exponents and Radical Functions

## Content Area: Mathematics

Course \& Grade Level: Algebra 2, 9-12

## Summary and Rationale

In this unit, students will study rational exponents and radical functions. The algebraic study of radical functions helps symbolize and generalize the rules of arithmetic. This builds a thorough understanding of our number system by applying the rules of arithmetic to the set of irrational numbers. Radical functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. By studying the domain, range and rate of change of radical functions, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions.

## Recommended Pacing

17 days

## New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning |
| Standard: A-SSE.A Interpret the structure of expressions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 l | Interpret complicated expressions by viewing one or more of their parts as a single entity. For <br> example, interpret P(1+r) n as the product of P and a factor not depending on P |

Standard: A-SSE.B Write expressions in equivalent forms to solve problems

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| $3 c$ | Use the properties of exponents to transform expressions for exponential functions. For example the <br> expression 1.15t can be rewritten as (1.151/12) 12t $\approx 1.01212 t$ to reveal the approximate equivalent <br> monthly interest rate if the annual rate is $15 \%$ |

Standard: A-REI.B Represent and solve equations and inequalities graphically

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 11 | Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ <br> intersect are the solutions of the equation $f(x)=g(x) ;$ find the solutions approximately, e.g., using <br> technology to graph the functions, make tables of values, or find successive approximations. Include <br> cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and <br> logarithmic functions. |
| Standard: F-IF.C Analyze functions using different representations |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| $7 e$ | Graph exponential and logarithmic functions, showing intercepts and end behavior, and <br> trigonometric functions, showing period, midline, and amplitude. |


| 8b | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. <br> b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=(1.02) t, y=(0.97) t, y=(1.01) 12 t, y$ $=(1.2) t / 10$, and classify them as representing exponential growth or decay. |
| :---: | :---: |
| Standard: F-LE.B Construct and compare linear and exponential models and solve problems |  |
| 1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. <br> a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. <br> b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. <br> c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| 2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |
| 3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |
| 4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $a b c t=d$ where $a, c$, and $d$ are numbers and the base $b$ is 2,10 , or e; evaluate the logarithm using technology |
|  | New Jersey Student Learning Standards for English Language Arts Companion Standards |
| Standard: Science Key Ideas and Details |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| RST.6-8.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics |
| RST.6-8.7 | Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). |
| New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 9.4.5.CI. 3 | Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a). |
| 9.4.5.CT. 3 | Describe how digital tools and technology may be used to solve problems. |
| 9.4.5.CT. 4 | Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3). |
| New Jersey Student Learning Standards for Technology |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |
| 8.2 | All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment. |
| Interdisciplinary Standards Science |  |
| HS.LS. 1 | Science example: Use a spreadsheet or other technology to simulate the doubling in a process of cell division; graph the results; write an expression to represent the number of cells after a division in terms of the number of cells beforehand; express this in closed form as a population size in terms of |



|  | Innovative \& Practical Problem Solver | Self-Directed Learner |
| :--- | :--- | :--- | :--- |
| Resources |  |  |
| Core Text: Big Ideas Math, Algebra 2; Larson and Boswell |  |  |

## Unit 5: Exponential Functions

## Content Area: Mathematics

Course \& Grade Level: Algebra 2, 9-12

## Summary and Rationale

In this unit, students will study exponential functions. Exponential functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. Exponential functions are used to model growth and decay. By studying the domain, range and rate of change of an exponential function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions.

## Recommended Pacing

12 days

## New Jersey Student Learning Standards for Mathematics

Standard: Standards for Mathematical Practice

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning |
| Standard: A-SSE.A Interpret the structure of expressions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| $1 b$ | Interpret complicated expressions by viewing one or more of their parts as a single entity. For <br> example, interpret P(1+r) n as the product of P and a factor not depending on P |

Standard: A-SSE.B Write expressions in equivalent forms to solve problems

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 3 c | Use the properties of exponents to transform expressions for exponential functions. For example the <br> expression 1.15t can be rewritten as (1.151/12) 12t $\approx 1.01212 \mathrm{t}$ to reveal the approximate equivalent <br> monthly interest rate if the annual rate is $15 \%$ |
| Standard: A-REI.B Represent and solve equations and inequalities graphically |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 11 | Explain why the $x$-coordinates of the points where the graphs of the equations $\mathrm{y}=\mathrm{f}(\mathrm{x})$ and $\mathrm{y}=\mathrm{g}(\mathrm{x})$ <br> intersect are the solutions of the equation $\mathrm{f}(\mathrm{x})=\mathrm{g}(\mathrm{x}) ;$ find the solutions approximately, e.g., using <br> technology to graph the functions, make tables of values, or find successive approximations. Include <br> cases where $f(\mathrm{x})$ and/or $\mathrm{g}(\mathrm{x})$ are linear, polynomial, rational, absolute value, exponential, and <br> logarithmic functions. |
| Standard: F-IF.C Analyze functions using different representations |  |


| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 7 e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and <br> trigonometric functions, showing period, midline, and amplitude. |
| 8 Bb | Write a function defined by an expression in different but equivalent forms to reveal and explain <br> different properties of the function. <br> b. Use the properties of exponents to interpret expressions for exponential functions. For <br> example, identify percent rate of change in functions such as y $=(1.02) \mathrm{t}, \mathrm{y}=(0.97) \mathrm{t}, \mathrm{y}=(1.01) 12 \mathrm{t}, \mathrm{y}$ <br> = (1.2)t/10, and classify them as representing exponential growth or decay. |
| Standard: F-LE.B Construct and compare linear and exponential models and solve problems |  |$|$| Distinguish between situations that can be modeled with linear functions and with exponential |
| :--- |
| functions. |
| a. Prove that linear functions grow by equal differences over equal intervals, and that exponential |
| functions grow by equal factors over equal intervals. |
| b. Recognize situations in which one quantity changes at a constant rate per unit interval relative |
| to another. |
| c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit |
| interval relative to another. |


| HS.LS. 1 | Science example: Use a spreadsheet or oth division; graph the results; write an expres terms of the number of cells beforehand; time. Discuss real-world factors in the situa over time. NGSS Appendix L | nology to simulate the doubling in a process of cell epresent the number of cells after a division in his in closed form as a population size in terms of t lead to deviation from the exponential model |
| :---: | :---: | :---: |
| Instructional Focus |  |  |
| Unit Enduring Understandings |  |  |
| - Mathematicians use algebraic and graphical representations to generalize patterns and relationships <br> - Inverse operations allow us to state mathematical facts in a variety of ways. <br> - The relationships between functions and their inverses relate to their domain and range <br> - Patterns and relationships can be represented graphically, numerically, symbolically, and verbally. Each representation has its advantages and disadvantages. <br> - Mathematicians use exponential equations to model, interpret and explain real-life phenomena. |  |  |
| Unit Essential Questions |  |  |
| - What are some of the characteristics of the graph of an exponential function? <br> - How do we model quantities that change over time by the same percentage? <br> - How do exponential functions model real-world problems and their solutions? <br> - How are expressions involving exponents related? <br> - What is the relationship between exponential functions? |  |  |
| Content Understandings |  |  |
| - If a function is invertible, then its domain becomes the range of that function's inverse and its range becomes the inverse's domain. <br> - Exponential functions can be used to model growth and decay. |  |  |
| Content Questions |  |  |
| - How does an exponential function model growth or decay? <br> - How do we graph exponential functions? <br> - How can you transform the graphs of exponential functions? |  |  |
| Objectives |  |  |
| Students will know: <br> - Vocabulary associated with exponents <br> - Properties of exponents <br> Students will be able to: <br> - Graph and translate exponential functions <br> - Evaluate expressions containing exponents <br> - Solve an exponential equation using various methods <br> - Determine domain and range of exponential functions <br> - Perform and write transformations of graphs of exponential functions. <br> - Model real world situations with exponential and use them to make predictions <br> - Use technology to determine the exponential regression equation for a given set of data <br> - Analyze rate of change to determine if data is best represented by a linear, exponential, or quadratic model |  |  |
| Evidence of Learning |  |  |
| Assessment |  |  |
| Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data. |  |  |
| Competencies for $\mathbf{2 1 ~}^{\text {st }}$ Century Learners |  |  |
| Collaborative Team Member |  | Effective Communicator |
| Globally Aware, Active, \& Responsible Student/Citizen |  | Information Literate Researcher |


|  | Innovative \& Practical Problem Solver |  | Self-Directed Learner |
| :--- | :--- | :--- | :--- |
| Resources |  |  |  |
| Core Text: Big Ideas Math, Algebra 2; Larson and Boswell |  |  |  |

## Unit 6: Rational Functions

| Content Area: Mathematics |  |
| :---: | :---: |
| Course \& Grade Level: Algebra 2, 9-12 |  |
|  | Summary and Rationale |
| In this unit, students will study rational functions. They will expand upon their knowledge of functions by exploring the ratio of two polynomials. The algebraic study of rational functions enables mathematicians to symbolize and generalize the rules of arithmetic. This builds a thorough understanding of our number system and the connectedness of the discipline of mathematics. The concept of variation is also explored. Students will use patterns to generalize how one variable changes based on other variables. |  |
| Recommended Pacing |  |
| 18 days |  |
| New Jersey Student Learning Standards for Mathematics |  |
| Standard: Standards for Math Practice |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning |
| Standard: A-APR.D Rewrite rational expressions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 6 | Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. |
| 7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| Standard: A-REI.A Understand solving equations as a process of reasoning and explain the reasoning |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| 2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise |
| Standard: F-IF.C Analyze functions using different representations |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior |
| New Jersey Student Learning Standards for English Language Arts Companion Standards |  |
| Stand | ence Key Ideas and Details |


| CPI \# | Cumulative Progress Indicator (CPI) |
| :---: | :---: |
| RST.6-8.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics |
| RST.6-8.7 | Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table). |
| New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 9.4.5.CI. 3 | Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a). |
| 9.4.5.CT. 3 | Describe how digital tools and technology may be used to solve problems. |
| 9.4.5.CT. 4 | Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3). |
| New Jersey Student Learning Standards for Technology |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |
| 8.2 | All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment. |
| Instructional Focus |  |
| Unit Enduring Understandings |  |
| - Rational equations are used to model, understand and explain real-life situations <br> - Non Vertical asymptotes model the end behavior of a graph and, therefore, can be crossed <br> - Domain restrictions affect the solutions of an algebraic equation. <br> - Restrictions in the domain of a rational function result in discontinuities |  |
| Unit Essential Questions |  |
| - What kinds of asymptotes are possible for a rational function? <br> - What are the advantages and disadvantages of various, equivalent forms of rational expressions? <br> - What does the factored form of an equation tell us about its graph? <br> - How do rational equations model real world relationships between variables? <br> - When two quantities vary, how do they relate to their constant of variation? |  |
| Content Understandings |  |
| - The vertical asymptotes of a rational function occur at the zeros of the polynomial in the denominator, as long as those zeros are not zeros of the polynomial in the numerator. <br> - A horizontal asymptote models a constant end behavior of a rational function <br> - Extraneous solutions occur when eliminating the common denominator, thus changing the set of possible solutions. |  |
| Content Questions |  |
| - Why do the graphs of rational functions differ visually from the parent function, in comparison to other types of functions? <br> - How do domain restrictions affect solutions to rational equations? <br> - Are a rational expression and its simplified form equivalent? Does $\left(x^{2}-1\right) /(x-1)$ equal $x+1$ ? |  |
| Objectives |  |
| Students will know: <br> - Vocabulary associated with rational functions and expressions <br> - Horizontal asymptotes in relation to end behavior <br> - Domain restrictions |  |

## Students will be able to:

- Graph rational functions using a calculator
- Add, subtract, multiply and divide rational expressions
- Simplify complex fractions
- Determine domain and range of rational functions
- Solve rational equations
- Utilize variation to model real world situations and use the model to make predictions
- Utilize rational functions to model real world situations and use the model to make predictions


## Evidence of Learning

## Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Competencies for $21^{\text {st }}$ Century Learners

| $x$ | Collaborative Team Member | X | Effective Communicator |
| :--- | :--- | :--- | :--- |
| x | Globally Aware, Active, \& Responsible Student/Citizen | x | Information Literate Researcher |
| x | Innovative \& Practical Problem Solver | x | Self-Directed Learner |
| Resources |  |  |  |
| Core Text: Big Ideas Math, Algebra 2; Larson and Boswell |  |  |  |

## Optional Unit 7: Probability

## Content Area: Mathematics <br> Course \& Grade Level: Algebra 2, 9-12

## Summary and Rationale

Probability helps to analyze the chance of events occurring and provides a framework with which to make decisions about future events. To determine probability of a particular event, students will also need to be aware of a variety of approaches for counting outcomes (Counting Principle, permutations, distinguishable permutations, combinations). Conditional probability will also be emphasized to help students reason about cause and effect and serve as an introduction to principles of experimental analysis.

## Recommended Pacing

13 days

## New Jersey Student Learning Standards for Mathematics

Standard: Standards for Math Practice

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning |
| Standard: S-CP.A Understand independence and conditional probability and use them to interpret data |  |


| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or <br> categories) of the outcomes, or as unions, intersections, or complements of other events ("or," <br> "and," "not"). |
| 2 | Understand that two events A and B are independent if the probability of A and B occurring together <br> is the product of their probabilities, and use this characterization to determine if they are <br> independent. |
| 3 | Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence <br> of A and B as saying that the conditional probability of A given B is the same as the probability of A, <br> and the conditional probability of B given A is the same as the probability of B. |
| 4 | Construct and interpret two-way frequency tables of data when two categories are associated with <br> each object being classified. Use the two-way table as a sample space to decide if events are <br> independent and to approximate conditional probabilities. For example, collect data from a random <br> sample of students in your school on their favorite subject among math, science, and English. <br> Estimate the probability that a randomly selected student from your school will favor science given <br> that the student is in tenth grade. Do the same for other subjects and compare the results. |
| 5 | Recognize and explain the concepts of conditional probability and independence in everyday <br> language and everyday situations. For example, compare the chance of having lung cancer if you are <br> a smoker with the chance of being a smoker if you have lung cancer. |


| Standard: S-CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability <br> model |  |
| :--- | :--- |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 6 | Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, <br> and interpret the answer in terms of the model. |
| 7 | Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of <br> the model. |
| 8 | Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B\|A) = <br> P(B)P(AlB), and interpret the answer in terms of the model. |
| 9 | Use permutations and combinations to compute probabilities of compound events and solve <br> problems |
| New Jersey Student Learning Standards for English Language Arts |  |
| Companion Standards |  |

- How can we use modeling to form a prediction?


## Content Understandings

- Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events
- Permutations and combinations are utilized to compute probabilities of compound events and to solve real-life problems
- Two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities
- The conditional probability of $A$ given $B$ is represented as $P(A$ and $B) / P(B)$
- Factorial notation ( $n$ !) is used to represent the number of ways $n$ things can be ordered.
- As a special case, the value of 0 ! is defined to be 1


## Content Questions

- How can a tree diagram help you visualize the number of ways in which two or more events can occur?
- How can you list the possible outcomes in the sample space of an experiment?
- How can you determine whether order is important for a given event?
- How can you determine when it is appropriate to use a permutation as opposed to a combination?
- If using the same values for $n$ and $r$, why are there more permutations than combinations?
- How can you determine whether two events are independent or dependent?
- What is the difference between experimental and theoretical probability?
- What is a simulation and how can it be useful?
- When would it be appropriate to use a two-way table?
- How can a two-way table be helpful when determining probabilities?
- What is the fewest number of pieces of information needed to complete a two-category table?
- How does calculating probability between mutually exclusive events compare to probability of events with shared outcomes?


## Objectives

Students will know:

- The Fundamental Counting Principle
- Factorial notation and how to simplify expressions containing ratio of factorial expressions efficiently
- How to find probabilities of independent and dependent events
- How to use conditional relative frequencies to find conditional probabilities
- How to use the formulas for the number of permutations and the number of combinations

Students will be able to:

- Use tree diagrams and the fundamental counting principle to represent the number of possible outcomes
- Use permutation and combination formulas to find probabilities
- Identify distinguishable permutations
- Determine the theoretical and experimental probability of an event
- Determine a sample space, find theoretical and experimental probabilities
- Determine whether events are independent/dependent and find probabilities
- Find conditional probabilities
- Find probabilities of compound events (overlapping/mutually exclusive)
- Construct and interpret probability distributions and binomial distributions
- Make and interpret two way tables
- Find probabilities of disjoint and overlapping event


## Evidence of Learning

## Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Competencies for $21^{\text {st }}$ Century Learners

|  | Collaborative Team Member |  | Effective Communicator |
| :--- | :--- | :--- | :--- |
|  | Globally Aware, Active, \& Responsible Student/Citizen |  | Information Literate Researcher |
|  | Innovative \& Practical Problem Solver | Resources |  |
| Self-Directed Learner |  |  |  |
| Core Text: Big Ideas Math, Algebra 2; Larson and Boswell |  |  |  |

