West Windsor-Plainsboro Regional School District
Advanced Algebra 2
August 2022

Unit 1: Linear and Absolute Value Functions	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
This unit will focus on the study of functions and relations. Function notation is used to describe relationships in terms of a dependent and independent variable. By studying the characteristics of a function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions. Furthermore, a deep understanding of functions is a crucial part of the foundation for higher level mathematics and physical sciences. Additionally, students will study linear and absolute value equations. Linear functions are used to describe relationships that have a constant rate of change in terms of a dependent and independent variable. The algebraic study of equations helps mathematicians symbolize and generalize the rules of arithmetic. Students apply their understanding of functions to a variety of situations in order to analyze and explain different numerical relationships.	
Recommended Pacing	
17 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Math Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: A-REI.C Solve systems of equations	
CPI #	**Cumulative Progress Indicator (CPI)**
5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Standard: A-REI.D Represent and solve equations and inequalities graphically	
CPI #	**Cumulative Progress Indicator (CPI)**
10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Standard: F-IF.A Understand the concept of a function and use function notation	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context
Standard: F-IF.B Interpret functions that arise in applications in terms of the context	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
Standard: F-IF.C Analyze functions using different representations	
7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima
9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
Standard: F-BF.A Build a function that models a relationship between two quantities	
1	Write a function that describes a relationship between two quantities.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).
Interdisciplinary Standards Science	
MS-PS3-1	Construct and interpret graphical displays of data to identify linear and nonlinear relationships.
Instructional Focus	
Unit Enduring Understandings	
• Functions model real world situations using various equations, graphs and tables. These models help mathematicians understand and explain real life data. • Mathematical solutions do not always represent a real or possible solution. • There are sets of functions, called families, in which each function is a transformation of a specific function called the parent.	
Unit Essential Questions	
• How can mathematicians model complex real world situations?	

- What is a function and why are they important when describing data?
- When is the mathematical solution to a problem not a viable solution?
- What does a graph tell you about a function?

Content Understandings

- Functions that possess a constant rate of change are classified as linear.
- There are special cases when solving equations that result in one solution, no solution, extraneous solutions, infinite solutions or all real number solutions.
- Functions can be classified based on rate of change.
- The properties of functions and function operations are used to model and analyze real world applications and quantitative relationships.
- Classifying different types of data allows us to better understand and analyze it, and one such form of identification is discrete vs continuous data. Functions can be discrete or continuous.
- New functions can be built from existing functions by applying transformations.

Content Questions

- How do the graphs of y = f(x) + k , y=f(x-h), y=f(-x) and y=kf(x) [for positive and negative values of k] compare to the parent function *f*?
- What is a reasonable domain for a given function?
- What is function notation and why is it used?
- How are relationships depicted or communicated?
- What is the difference between a function and a relation?
- How can you determine a linear function by looking at a graph, a table of values or an algebraic equation?
- What are the characteristics of some of the basic parent functions?
- What is the most efficient way to graph given a particular function?

Objectives

Students will know:

- The association between relations and functions
- The difference between domain and range
- The properties of transformations of functions
- The sequencing of a list of transformations could result in different graphs

Students will be able to:

- Compare functions that are represented in different forms (i.e. a table, a rule, a verbal description, a graph and a set of ordered pairs)
- Identify the domain and range from a graph and/or an equation using interval notation.
- Describe the transformation (rigid and non-rigid) of a parent function that would yield the graph of a given function.
- Write a transformed function given a list of transformations to a parent function.
- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another and apply this concept to solve real life applications.
- Graph the equation of a line given in standard, slope intercept or point slope form.
- Find the equation of a line given two points or a point and its slope.
- Determine if two lines are parallel or perpendicular.
- Use linear functions to solve real life application
- Solve absolute value equations.
- Graph absolute value functions using transformations.
- Analyze piecewise functions.
- Graph relations and be able to determine whether or not the relation is a function.
- Utilize technology to graph relations (Desmos, TI-84, etc.).
- Use f(x) terminology to evaluate functions.
- Utilize variables to represent unknown quantities in real world situations.
- Identify all values that make an equation a true statement.

Evidence of Learning
Assessment
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized tests and NJSLA data.
Resources
Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

Unit 2: Quadratics	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
This unit will focus on the study of quadratic functions. Quadratic functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. These functions can be visually represented by a parabola. By studying the domain, range and rate of change of a quadratic function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions. There are common forms in which a quadratic can be written and each gives information about the graph and behavior of the function. Understanding the relationships between the characteristics of a quadratic and its equation will forge the connections between the method of graphing a quadratic function based on its algebraic form. Solving quadratic equations leads to zeros of a quadratic function which in turn can translate to the x-intercepts of its graph. Solving techniques will be explored with a new emphasis on including the possibility of complex numbers. In this unit, the solutions of quadratics will no longer be restricted to real numbers.	
Recommended Pacing	
22 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Math Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: N-CN.A Perform arithmetic operations with complex numbers	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Know there is a complex number i such that i 2 = –1, and every complex number has the form a + bi with a and b real.
2	Use the relation i 2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers
7	Solve quadratic equations with real coefficients that have complex solutions.
Standard: A-CED.A Create equations that describe numbers or relationships	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions
Standard: A-REI.B Solve equations and inequalities in one variable	
CPI #	**Cumulative Progress Indicator (CPI)**
4	Solve quadratic equations in one variable. .

	a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
Standard: F-IF.C Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
8a	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context
9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).
Interdisciplinary Standards Science	
HS.PS.2	Science examples: (1) Informally fit a quadratic function to the position-time data for a cart that rolls up an incline (slowing as it climbs, then reversing direction and speeding up as it descends). Use the algebraic expression for the fitted function to determine the magnitude of the cart's acceleration and initial speed. Over several trials, graph various quantities (such as acceleration vs. angle, or peak displacement vs. initial speed squared), and interpret the results. (2) Calculate and interpret the average speed of a moving object by using data from a distance-time graph. NGSS Appendix L
Instructional Focus	
Unit Enduring Understandings	
• Mathematicians use algebraic and graphical representations to generalize patterns and relationships • Real world situations involving quadratic relationships can be modeled and solved using multiple representations. Each representation has its advantages and disadvantages.	
Unit Essential Questions	
• How can we use mathematical language to describe non-linear change?	

- How can we use mathematical models to describe physical and scientific relationships?
- When is an estimate an appropriate solution in a real-life situation?

Content Understandings

- A graphical representation of quadratic data can be used to analyze and extrapolate pertinent information.
- Quadratic equations and functions can be solved using a variety of methods
- Recognizing similarities between a quadratic equation and the graph of the corresponding function
- X-intercepts are potential boundaries between negative and positive outputs and therefore are classified as "critical numbers."
- Analyzing data to determine a constant rate of change or second rate of change helps determine if data is linear or quadratic
- The imaginary unit i was invented to allow us to express complex solutions to quadratic equations
- The solution to a system is graphically represented as the intersection of the two functions

Content Questions

- Which is the most efficient method for solving a quadratic equation?
- What are the advantages and disadvantages of expressing a quadratic in various forms?
- When are some forms more appropriate than others?
- How are quadratic functions related to each other?
- How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?
- What do the solutions of a quadratic equation tell you about its graph?
- What are the subsets of the set of complex numbers?
- How can we use quadratic equations to model and interpret real-life situations?
- How can you determine the number of solutions of a system of equations?

Objectives

Students will know:

- Characteristics of quadratic functions.
- A parabola is the graphical representation of a quadratic function
- Procedures for solving quadratic equations and functions
- All the forms of a quadratic function: standard, vertex, intercept form.
- How the parameters for each form of the quadratic function affects the attributes of its corresponding graph
- The Quadratic Formula
- How transformations affect the parent function.

Students will be able to:

- Identify characteristics of a quadratic function
- Graph a quadratic given multiple transformations of the parent function
- Determine transformations from a quadratic equation.
- Graph quadratic functions through standard, vertex, and intercept form
- Solve quadratic equations through the methods of factoring, square roots, completing the square, and the quadratic formula
- Algebraically convert between quadratic forms
- Determine maximum/minimum of a parabola through various methods
- Find the x-intercepts of a quadratic function through various methods
- Choose and use the best method of solution for a quadratic equation
- Determine the discriminant and the nature of the roots of a quadratic equation
- Relate the nature of the solutions to the discriminant and the graph
- Solve a quadratic equation whose solutions are complex numbers
- Simplify complex numbers
- Add, subtract, multiply complex numbers
- Write a quadratic equation, given particular information about the function.

• Utilize quadratic equations to model real world situations and use them to make predictions • Use a graphing calculator to determine the quadratic regression equation of sets of data • Solve a nonlinear system.
Evidence of Learning
Assessment
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Resources
Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

Unit 3: Polynomial Functions	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
This unit will focus on the study of polynomial functions. Polynomial functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. By studying the domain, range and rate of change of a polynomial function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions. Students will extend previous connections made between quadratic functions and their graphs. The graphs of polynomial functions have attributes (end behavior, x-intercept behavior, turning points) that are determined by key components of the function itself. One of the key components to analyzing a polynomial function's graph is its x-intercepts which are directly related to the zeros (or roots) of the function. In this unit, students will learn that finding a polynomial function's factored form is the most convenient approach to finding these zeros. However, when this approach is not feasible, tools such as synthetic division and The Rational Roots Theorem can help with the search for zeros.	
Recommended Pacing	
20 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: A-APR.B Understand the relationship between zeros and factors of polynomials	
CPI #	**Cumulative Progress Indicator (CPI)**
2	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Standard: F-IF.B Interpret functions that arise in applications in terms of the context	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
Standard: F-IF.C Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**

7c	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).

Instructional Focus

Unit Enduring Understandings

- A polynomial function has distinguishing behaviors. Its algebraic form gives information about its graph and its graph gives information about its algebraic form.
- The degree of a polynomial function tells you how many roots the function has.
- If $x - k$ is a factor of a polynomial function $f(x)$, then *k* is a solution to when $f(x) = 0$, *k* is a zero of $f(x)$ and $(k, 0)$ is an *x*-intercept of the graph of $f(x)$ when *k* is a real number.
- Polynomial functions allow us to model real world applications found in various mathematical disciplines.

Unit Essential Questions

- How do the components of a polynomial function affect the characteristics of its graph?
- For a polynomial function, how are factors, zeros and x-intercepts related?
- How does the multiplicity of a root affect the behavior of the graph at the related x-intercept?
- What tools are there to help us search efficiently for zeros of a polynomial function?
- How do polynomial functions model real-world problems and their solutions?

Content Understandings

- A polynomial is a monomial or a sum of monomials; each monomial is a numeral, variable or a product of variables. Thus the form of a polynomial function is $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x + a_0$ where each power of *x* is a whole number and each coefficient is real.
- Polynomial functions as graphs must be continuous (no breaks, gaps or holes). Their domain is always "all real numbers". Their turns are always rounded.
- For polynomial function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x + a_0$, the end behavior is determined by the sign of a_n and whether *n* is even or odd. There are four possible end behaviors to consider.
- Identifying extrema (mins/maxes) and increasing/decreasing sections of a polynomial function's graph help us to describe its distinguishing characteristics.
- You can divide polynomials using steps that are similar to the long division steps that you use to divide whole numbers.

- Turning points are extrema that separate increasing and decreasing sections of a polynomial function's graph. There are at most $n - 1$turning points for the graph of a polynomial function of degree *n*.

Content Questions

- What is the difference between degree and exponent?
- Why must rational roots of a polynomial function be in the form of a factor of the constant over a factor of the leading coefficient?
- How does polynomial long division resemble numerical long division? How can this connection be used to understand the Division Algorithm?
- Why does synthetic division work? When can it be used?
- What is the difference between turning points and inflection points?
- Using what you know about end behavior, can you draw a quintic with an odd number of turning points?
- What are the benefits of synthetically dividing $x - k$ over evaluating $f(k)$ to determine if *k* is a root?

Objectives

Students will know:

- The number of turning points in the graph of a polynomial function will be at most 1 less than its degree
- Polynomial functions with an odd degree have an even number of turning points in its graph; those with an even degree have an odd number of turning points in its graph
- Synthetic division is a shorthand version of long division only to be used in the case of division between polynomials where a divisor is in the form of $x - k$
- Remainder Theorem
- Factor Theorem
- Rational Roots Theorem
- Irrational Conjugates Theorem
- Complex Conjugates Theorem
- Fundamental Theorem of Algebra and its corollary that a polynomial function of degree *n* must have *n* complex roots (including multiplicity)

Students will be able to:

- Identify the end behavior of a polynomial function based on its degree and the sign of its leading coefficient
- Identify the multiplicity of a root and use it to determine x-intercept behavior
- Sketch the graph of a polynomial function based on its end behavior and x-intercept behavior
- Identify increasing and decreasing regions in a polynomial function and express what portion(s) of the domain apply to these regions
- Identify relative and absolute extrema in the graph of a polynomial function and find extrema using technology
- Determine if a function is even or odd by the results of $f(-x)$or by analyzing symmetry in its graph (using Desmos)
- Use long division to divide polynomials and express their results using the Division Algorithm.
- Use synthetic division to test for zeros of a polynomial function
- Evaluate a function using synthetic division (Remainder Theorem)
- Factor polynomial functions completely using techniques from quadratics factoring, factoring by grouping, difference of squares
- Write a polynomial function of least degree with rational coefficients when given certain roots (including irrational and/or imaginary)
- Identify the conjugate of an irrational or complex number and determine the product of the conjugates using a difference of squares
- Describe transformations of polynomial functions
- Write transformations of polynomial functions
- Write polynomial functions for sets of points
- Use technology to find models for data sets whose plots appear to be polynomial-based

Evidence of Learning
Assessment
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Resources
Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

Unit 4: Rational Exponents and Radical Functions	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
In this unit, students will study rational exponents and radical functions. The algebraic study of radical functions helps symbolize and generalize the rules of arithmetic. This builds a thorough understanding of our number system by applying the rules of arithmetic to the set of irrational numbers. Radical functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. By studying the domain, range and rate of change of radical functions, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions.	
Recommended Pacing	
18 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: A-SSE.A Interpret the structure of expressions	
CPI #	**Cumulative Progress Indicator (CPI)**
1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r) n as the product of P and a factor not depending on P
Standard: A-SSE.B Write expressions in equivalent forms to solve problems	
CPI #	**Cumulative Progress Indicator (CPI)**
3c	Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12) 12t ≈1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%
Standard: A-REI.B Represent and solve equations and inequalities graphically	
CPI #	**Cumulative Progress Indicator (CPI)**
11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Standard: F-IF.C Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8b	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.
Standard: F-LE.B Construct and compare linear and exponential models and solve problems	
1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).
Interdisciplinary Standards Science	
HS.LS.1	Science example: Use a spreadsheet or other technology to simulate the doubling in a process of cell division; graph the results; write an expression to represent the number of cells after a division in terms of the number of cells beforehand; express this in closed form as a population size in terms of time. Discuss real-world factors in the situation that lead to deviation from the exponential model over time. NGSS Appendix L
Instructional Focus	
Unit Enduring Understandings	
● Irrational equations are used to model, understand and explain real-life situations ● Mathematicians use algebraic and graphical representations to generalize patterns and relationships ● Inverse operations allow us to state mathematical facts in a variety of ways.	

- The relationships between functions and their inverses relate to their domain and range
- Patterns and relationships can be represented graphically, numerically, symbolically, and verbally. Each representation has its advantages and disadvantages.
- Mathematicians use radical equations to model, interpret and explain real-life phenomena.

Unit Essential Questions

- How do we use irrational equations to model, analyze, understand and explain real world situations?
- What are some of the characteristics of the graph of a radical function?
- How do we model quantities that change over time by the same percentage?
- What are the characteristics of inverse functions?
- How do radical functions model real-world problems and their solutions?

Content Understandings

- If a function is invertible, then its domain becomes the range of that function's inverse and its range becomes the inverse's domain.
- Fluency between radical and rational exponential form allows us to simplify efficiently
- Simplifying a radical expression to its simplest form allows us to combine like terms

Content Questions

- When and how do we simplify expressions containing rational exponents?
- How do we write rational exponents as radicals?
- How do we write radicals using rational exponents?
- How do we solve equations containing radicals?
- Why is it that when squaring each side of an equation, the result is not equivalent to the original?
- How is the domain affected when performing operations with functions?

Objectives

Students will know:

- Vocabulary associated with rational exponents and radical functions
- Equations with radicals can be graphed as functions

Students will be able to:

- Simplify rational exponents and radical expressions when appropriate
- Graph radical equations
- Solve radical equations
- Perform function operations algebraically and graphically
- Find the inverse equation when given the equations of a function
- Graph the inverse of a function
- Determine domain and range of inverse functions

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

Unit 5: Exponential and Logarithmic Functions	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
In this unit, students will study exponential and logarithmic functions. Exponential and logarithmic functions are used to describe relationships that have a variable rate of change, in terms of a dependent and independent variable. Exponential functions are used to model growth and decay. By studying the domain, range and rate of change of an exponential or logarithmic function, mathematicians can describe and analyze relationships. This understanding can provide the foundation to make decisions and reasonable predictions.	
Recommended Pacing	
15 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: A-SSE.A Interpret the structure of expressions	
CPI #	**Cumulative Progress Indicator (CPI)**
1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r) n as the product of P and a factor not depending on P
Standard: A-SSE.B Write expressions in equivalent forms to solve problems	
CPI #	**Cumulative Progress Indicator (CPI)**
3c	Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12) 12t ≈1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%
Standard: A-REI.B Represent and solve equations and inequalities graphically	
CPI #	**Cumulative Progress Indicator (CPI)**
11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Standard: F-IF.C Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8b	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

	b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.
Standard: F-LE.B Construct and compare linear and exponential models and solve problems	
1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).
Interdisciplinary Standards Science	
HS.LS.1	Science example: Use a spreadsheet or other technology to simulate the doubling in a process of cell division; graph the results; write an expression to represent the number of cells after a division in terms of the number of cells beforehand; express this in closed form as a population size in terms of time. Discuss real-world factors in the situation that lead to deviation from the exponential model over time. NGSS Appendix L
Instructional Focus	
Unit Enduring Understandings	
● Mathematicians use algebraic and graphical representations to generalize patterns and relationships ● Inverse operations allow us to state mathematical facts in a variety of ways. ● The relationships between functions and their inverses relate to their domain and range ● Patterns and relationships can be represented graphically, numerically, symbolically, and verbally. Each representation has its advantages and disadvantages.	

- Mathematicians use exponential and logarithmic equations to model, interpret and explain real-life phenomena.

Unit Essential Questions

- What are some of the characteristics of the graph of an exponential function?
- How do we model quantities that change over time by the same percentage?
- What are the characteristics of inverse functions?
- What is a logarithm and how do mathematicians use them?
- How can you use properties of exponents to derive properties of logarithms?
- How do exponential functions model real-world problems and their solutions?
- How are expressions involving exponents and logarithms related?
- What is the relationship between logarithmic and exponential functions?
- What are some of the characteristics of the graph of a logarithmic function?
- How can you recognize polynomial and exponential models?
- Where does e occur naturally in real-life situations?

Content Understandings

- If a function is invertible, then its domain becomes the range of that function's inverse and its range becomes the inverse's domain.
- Exponential functions can be used to model growth and decay.
- The inverse function of the natural logarithmic function is called the natural exponential function.
- You can use logarithms to solve exponential equations; and conversely, you can use exponents to solve logarithmic properties
- Logarithms and exponents have corresponding properties.
- The inverse relationship between exponential and logarithmic functions can be used to graph logarithmic functions

Content Questions

- How does an exponential function model growth or decay?
- How do we graph exponential functions?
- How can you transform the graphs of exponential functions?
- How do we use the properties of logarithms to expand or condense expressions?
- How do we write an exponential equation in logarithmic form using the definition of a log?
- How do we write a logarithmic equation in exponential form using the definition of a log?
- How do you evaluate a logarithm without a calculator?
- What is the natural base e?

Objectives

Students will know:

- Vocabulary associated with exponents and logarithms
- Properties of exponents and logarithms
- Logarithms can be used to solve exponential equations
- The definition and use of the natural base e
- Characteristics of exponential and logarithmic graphs

Students will be able to:

- Recognize how transformations affect exponential functions
- Evaluate expressions containing exponents and logarithms
- Solve an exponential equation using various methods
- Utilize properties of logarithms to transform expressions and solve equations
- Determine domain and range of exponential, logarithmic and inverse functions
- Model real world situations with exponential functions and use them to make predictions
- Use technology to determine the exponential regression equation for a given set of data.
- Analyze rate of change to determine if data is best represented by a linear, exponential, or quadratic model

- Simplify natural logarithmic expressions   - Find the inverse of an exponential function and identify the domain and range
Evidence of Learning
Assessment
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Resources
Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

Unit 6: Sequences and Series	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
In this unit, students will study sequences and series. These are very powerful tools in mathematics, used for approximating functions. Particular series, including arithmetic and geometric series, are explored as discrete functions that model linear and exponential growth and decay. Students will further develop skills in pattern recognition and using multiple forms to represent the same mathematical idea.	
Recommended Pacing	
10 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Math Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: A-SSE.B Write expressions in equivalent forms to solve problems	
CPI #	**Cumulative Progress Indicator (CPI)**
4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
Standard: F-IF.A Understand the concept of a function and use function notation	
CPI #	**Cumulative Progress Indicator (CPI)**
3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
Standard: F-BF.A Build a function that models a relationship between two quantities	
CPI #	**Cumulative Progress Indicator (CPI)**
1a	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context
2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).

Instructional Focus

Unit Enduring Understandings

- Sequences and series can be used to model discrete mathematical relations and predict future values.
- Arithmetic sequences can be used to model discrete linear relationships, while geometric sequences can be used to model exponential relationships
- Explicit rules for sequences create discrete functions that represent sequences.
- Recursive rules for sequences create an algorithm to find a specific term in sequences based on previous terms.

Unit Essential Questions

- How are patterns, relations, and functions used as tools to best describe, analyze and explain real-life phenomena?
- Why do mathematicians represent the terms of a sequence explicitly? Recursively?
- How do mathematicians model arithmetic and geometric sequences and series? What is the purpose of these models?

Content Understandings

- The common difference is represented in the slope of the discrete linear function that represents an arithmetic sequence
- The common ratio is represented in the base of the discrete exponential function that represents a geometric sequence
- The partial sum of an arithmetic series can be found
- An infinite sum of an arithmetic series is divergent
- The partial sum of a geometric series can be found
- An infinite sum of a geometric series is convergent if the common ratio is between -1 and 1
- The lower limit of a summation can vary, although it is usually 0 or 1

Content Questions

- How do the characteristics of an arithmetic sequence relate to a discrete linear function?
- How do the characteristics of a geometric sequence relate to a discrete exponential function?

Objectives

Students will know:

- Vocabulary associated with sequences and series
- Arithmetic and geometric sequences and series
- Partial sum vs. Infinite sum
- Explicit and recursive rules for sequences

Students will be able to:

- Describe rules (explicit and/or recursive) for the nth term of a sequence
- Determine whether a sequence is arithmetic, geometric, or neither
- Insert a number of arithmetic means into a given sequence
- Insert a number of geometric means into a given sequence
- Evaluate a partial sum through various methods
- Write S_n using $\sum$ notation
- Find sums of infinite geometric series

● Use arithmetic or geometric sequences or series as mathematical models and the utilize the models to make predictions ● Rewrite simple repeating decimals as an infinite sum
Evidence of Learning
Assessment
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Resources
Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

Unit 7: Rational Functions	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
In this unit, students will study rational functions. They will expand upon their knowledge of functions by exploring the ratio of two polynomials. The algebraic study of rational functions enables mathematicians to symbolize and generalize the rules of arithmetic. This builds a thorough understanding of our number system and the connectedness of the discipline of mathematics. The concept of variation is also explored. Students will use patterns to generalize how one variable changes based on other variables.	
Recommended Pacing	
10 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Math Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: A-APR.D Rewrite rational expressions	
CPI #	**Cumulative Progress Indicator (CPI)**
6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Standard: A-REI.A Understand solving equations as a process of reasoning and explain the reasoning	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise
Standard: F-IF.C Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior
New Jersey Student Learning Standards for English Language Arts **Companion Standards**	
Standard: Science Key Ideas and Details	

CPI #	Cumulative Progress Indicator (CPI)
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).

Instructional Focus

Unit Enduring Understandings

- Rational equations are used to model, understand and explain real-life situations
- Non Vertical asymptotes model the end behavior of a graph and, therefore, can be crossed
- Domain restrictions affect the solutions of an algebraic equation.
- Restrictions in the domain of a rational function result in discontinuities

Unit Essential Questions

- What kinds of asymptotes are possible for a rational function?
- What are the advantages and disadvantages of various, equivalent forms of rational expressions?
- What does the factored form of an equation tell us about its graph?
- How do rational equations model real world relationships between variables?
- When two quantities vary, how do they relate to their constant of variation?

Content Understandings

- The vertical asymptotes of a rational function occur at the zeros of the polynomial in the denominator, as long as those zeros are not zeros of the polynomial in the numerator.
- A horizontal asymptote models a constant end behavior of a rational function
- Extraneous solutions occur when eliminating the common denominator, thus changing the set of possible solutions.

Content Questions

- Why do the graphs of rational functions differ visually from the parent function, in comparison to other types of functions?
- How do domain restrictions affect solutions to rational equations?
- Are a rational expression and its simplified form equivalent? Does $(x^2-1)/(x-1)$ equal $x+1$?

Objectives

Students will know:

- Vocabulary associated with rational functions and expressions
- Horizontal asymptotes in relation to end behavior
- Domain restrictions

Students will be able to:

- Graph rational functions
- Add, subtract, multiply and divide rational expressions
- Simplify complex fractions
- Determine domain and range of rational functions
- Solve rational equations
- Utilize variation to model real world situations and use the model to make predictions
- Utilize rational functions to model real world situations and use the model to make predictions

Evidence of Learning

Assessment
Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.
Resources
Core Text: Big Ideas Math, Algebra 2; Larson and Boswell

Unit 8: Probability	
Content Area: Mathematics	
Course & Grade Level: Advanced Algebra 2, 9-12	
Summary and Rationale	
Probability helps to analyze the chance of events occurring and provides a framework with which to make decisions about future events. To determine probability of a particular event, students will also need to be aware of a variety of approaches for counting outcomes (Counting Principle, permutations, distinguishable permutations, combinations). Connections will be made between combinations, Pascal's Triangle and binomial expansion. Conditional probability will also be emphasized to help students reason about cause and effect and serve as an introduction to principles of experimental analysis.	
Recommended Pacing	
11 days	
New Jersey Student Learning Standards for Mathematics	
Standard: Standards for Math Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning
Standard: S-CP.A Understand independence and conditional probability and use them to interpret data	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Standard: S-CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model	
CPI #	**Cumulative Progress Indicator (CPI)**
6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9	Use permutations and combinations to compute probabilities of compound events and solve problems
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Science Key Ideas and Details	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.6-8.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics
RST.6-8.7	Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.5.CT.3	Describe how digital tools and technology may be used to solve problems.
9.4.5.CT.4	Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).
9.4.8.IML.3	Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping (e.g., 6.SP.B.4, 7.SP.B.8b).
9.4.8.IML.4	Ask insightful questions to organize different types of data and create meaningful visualizations.

Instructional Focus

Unit Enduring Understandings

- Probability is about prediction over the long term rather than predictions of individual events.
- There are patterns of chance in numerical outcomes that statisticians use to predict the future.
- Discussing and determining the likelihood of an event relies on recognizing when to utilize the fundamental counting principle and recognizing which type of probability we are working with.

Unit Essential Questions

- How can you list the possible outcomes in the sample space of an experiment?
- How is probability related to real world events?
- How can experimental and theoretical probabilities be used to make predictions or draw conclusions?
- What affects the probability that a given event will occur?
- How can we use modeling to form a prediction?

Content Understandings

- Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events
- Permutations and combinations are utilized to compute probabilities of compound events and to solve real-life problems
- Two events A and B are independent if the probability of A and B occurring together is the product of their

probabilities
- The conditional probability of A given B is represented as P(A and B)/P(B)
- Factorial notation (n!) is used to represent the number of ways n things can be ordered.
- As a special case, the value of 0! is defined to be 1

Content Questions

- What is the relationship between Pascal's Triangle, combinations and binomial expansion?
- How can a tree diagram help you visualize the number of ways in which two or more events can occur?
- How can you list the possible outcomes in the sample space of an experiment?
- How can you determine whether order is important for a given event?
- How can you determine when it is appropriate to use a permutation as opposed to a combination?
- If using the same values for n and r, why are there more permutations than combinations?
- How can you determine whether two events are independent or dependent?
- What is the difference between experimental and theoretical probability?
- What is a simulation and how can it be useful?
- When would it be appropriate to use a two-way table?
- How can a two-way table be helpful when determining probabilities?
- What is the fewest number of pieces of information needed to complete a two-category table?
- How does calculating probability between mutually exclusive events compare to probability of events with shared outcomes?

Objectives

Students will know:

- The Fundamental Counting Principle
- Factorial notation and how to simplify expressions containing ratio of factorial expressions efficiently
- Pascal's Triangle
- How to find probabilities of independent and dependent events
- How to use conditional relative frequencies to find conditional probabilities
- How to use the formulas for the number of permutations and the number of combinations
- That you can use combinations and the Binomial Theorem to expand binomials

Students will be able to:

- Use tree diagrams and the fundamental counting principle to represent the number of possible outcomes
- Use permutation and combination formulas to find probabilities
- Identify distinguishable permutations
- Use combinations and the Binomial Theorem to expand binomials.
- Determine the theoretical and experimental probability of an event
- Determine a sample space, find theoretical and experimental probabilities
- Determine whether events are independent/dependent and find probabilities
- Find conditional probabilities
- Find probabilities of compound events (overlapping/mutually exclusive)
- Construct and interpret probability distributions and binomial distributions
- Make and interpret two way tables
- Find relative and conditional relative frequencies
- Use conditional relative frequencies to find conditional probabilities
- Find probabilities of disjoint and overlapping event

Evidence of Learning

Assessment

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of standardized test and NJSLA data.

Resources

Core Text: Big Ideas Math, Algebra 2; Larson and Boswell