# West Windsor-Plainsboro Regional School District
# Geometry Honors and Accelerated
# Curriculum Document

# Unit 1: Foundations in Geometry and Similarity

**Content Area:** Mathematics

**Course & Grade Level:** Geometry Honors and Accelerated/Ninth Grade

**Summary and Rationale**

In this unit, students will study the foundational information for the course: analytic Geometry, similarity and the tangent ratio. All further concepts will be developed building upon these three foundational ideas. The connections between algebra and geometry are made apparent at the inception of the course. Solving problems using similarity and tangent ratio will build the proportional reasoning that leads to success in the further study of mathematics.

Algebra is heavily integrated in the form of analytic geometry, which gives students another lense for analyzing or explaining geometric relationships. Students will use a rectangular coordinate system to verify geometric relationships, including properties and slopes of parallel and perpendicular lines. Students begin laying the groundwork for proofs in this unit with algebraic proofs of geometric properties. This will lay the cornerstone for logical reasoning and formal proofs in a future unit.

Students will use similar triangles to find missing segment lengths. They will explore the interrelationships between the trigonometric functions and use these ratios, along with the Pythagorean Theorem, to solve proportions, right triangles, given different initial information. Ratios in similar triangles will lead to the development and application of the tangent function.

The tangent function was developed from ratios within right triangles and is essential in modeling periodic behavior. Students will become adept at applying, manipulating, and developing relationships with tangents and arctangents in this unit. This unit is designed to expand upon right triangle similarity and lead into the exploration of the sine and cosine function in a future chapter.

**Recommended Pacing**

32 days

**New Jersey Student Learning Standards for Mathematics**

| **Standard: Standards for Mathematical Practice** | |
|---|---|
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics. |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning. |
| **Standard:** G-SRT.A Understand similarity in terms of similarity transformations. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. |
| **Standard:** G-SRT.B Prove theorems involving similarity. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |

| | |
|---|---|
| 4 | Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. |
| 5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. |
| **Standard:** G-SRT.C Define trigonometric ratios and solve problems involving right triangles. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. |
| 8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. |
| **Standard:** G-GPE.A Translate between the geometric description and the equation for a conic section. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. |
| **Standard:** G-GPE.B Use coordinates to prove simple geometric theorems algebraically. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 4 | Use coordinates to prove simple geometric theorems algebraically. |
| **New Jersey Student Learning Standards for English Language Arts Companion Standards** | |
| **Standard:** Science Key Ideas and Details | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| RST.9-10.3 | Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. |
| **Standard:** Science Craft and Structure | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| RST.9-10.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics. |
| **New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 9.4.12.CI.1 | Demonstrate the ability to reflect, analyze, and use creative skills and ideas. |
| 9.4.12.CT.2 | Explain the potential benefits of collaborating to enhance critical thinking and problem solving. |
| 9.4.12.TL.1 | Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task. |
| 9.4.12.TL.3 | Analyze the effectiveness of the process and quality of collaborative environments. |
| **New Jersey Student Learning Standards for Technology** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |
| **Interdisciplinary Standards: NGSS** | |
| MS-ESS1 | Earth's Place in the Universe<br>As part of this work, teachers should give students opportunities to use ratios and proportional relationships and use order of magnitude thinking. Science example: C Create scale-preserving descriptions such as, "If the solar system were shrunk down to the size of Earth, then Earth would shrink to the size of _____"; compute relevant scale factors and use them to determine a suitable object. (p. 25 NGSS Crosswalk) |

**Instructional Focus**

**Unit Enduring Understandings**

- Euclidean Geometry is a formal set of rules and theorems developed from three undefined terms and five postulates.
- There are many ways to communicate mathematical ideas deductively and effectively.
- Similarity is an extension of proportional reasoning that helps us measure indirectly.

**Unit Essential Questions**

- What's the difference between a postulate, property, and theorem?
- What happens if we change the Parallel postulate?
- How do geometric relationships help to solve problems and/or make sense of phenomena?
- How can we best represent geometric relationships?
- How can we measure the height of a tall building without physically measuring it?

**Content Understanding**

- Algebraic skills can be used to prove facts about geometric figures in a plane.
- Inductive and deductive are two types of reasoning.
- We prove figures similar so that we can measure geometric objects indirectly.
- The tangent ratio is a proportion that holds true for all angles of the same value in a right triangle.

**Content Questions**

- How can you find the midpoint and length of a line segment?
- How are similar polygons related?
- What proportionality relationships exist when a special segment or line intersects a triangle?
- How are the values of the tangent function of right triangles connected to the similarity of the right triangles?
- What is the main difference between the tangent function and its inverse?
- How can the tangent and inverse tangent function be used to solve right triangles and find unknown measurements?

**Objectives**

**Students will know:**

- **Undefined terms:** point, line, plane
- **Terms:** equidistant, distance, collinear, coplanar, intersection, segment, ray, angle, length, congruent, midpoint, bisector, postulate, theorem, ratio, proportion, coordinate proof, means, extremes, similar, scale factor (similarity ratio), reflexive property, symmetric property, transitive property, substitution property, distributive property, addition property of equalities, subtraction property of equalities, multiplication property of equalities, division property of equalities, tangent and arctan function
- **Postulates:** Line Postulate, Parallel Postulate, Perpendicular Postulate, Linear-Pair Postulate, Segment Addition Postulate, Angle Addition Postulate, Corresponding Angle (CA) Postulate, Converse CA Postulate, SSS Postulate, ASA Postulate, SAS Postulate, AA Similarity Postulate
- **Theorems:** Line Intersection Theorem, SAS Similarity Theorem, SSS Similarity Theorem, Triangle Proportionality Theorem, Pythagorean Theorem, Converse of the Pythagorean Theorem, Theorems about the lengths of the sides of an acute/obtuse triangle, Theorems dealing with parallel lines and proportional segments, Theorems about the altitude drawn to the hypotenuse of a right triangle and the geometric mean

**Students will be able to:**

- Use knowledge of points, lines, and planes appropriately
- Apply concepts of collinearity and betweenness in problem solving
- Use deductive structure to develop proofs of basic geometric concepts
- Use geometry software to demonstrate conjectures of basic geometric concepts

- Solve problems and form predictions involving probability measures
- Explore the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius given by an equation
- Use similarity to solve authentic problems
- Apply the tangent and arctan function to solve application problems

**Evidence of Learning**

**Assessment**

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

**Resources**

**Core Text:** Geometry by Big Ideas Learning - Larson and Boswell 2022
**Suggested Resources:** Trapeze Education

# Unit 2: Logic and Proofs

**Content Area:** Mathematics

**Course & Grade Level:** Geometry Honors and Accelerated/Ninth grade

**Summary and Rationale**

In this unit, students are encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Although there are many types of geometry, our course is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). In this unit, students will begin to develop logical arguments into formal proofs. They will formalize their arguments using more precise definitions and developing careful explanations and proofs. Students will study logic to develop an understanding of Euclidean geometry as the product of a small set of axioms. The logical Laws of Inference will be used to draw conclusions from real world problems.

**Recommended Pacing**

17 days

**New Jersey Student Learning Standards for Mathematics**

**Standard: Standards for Mathematical Practice**

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics. |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning. |

**Standard:** G-CO.C Prove geometric theorems.

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 9 | Prove theorems about lines and angles. |

**New Jersey Student Learning Standards for English Language Arts Companion Standards**

**Standard:** Science Key Ideas and Details

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| RST.9-10.3 | Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. |

**Standard:** Science Craft and Structure

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| RST.9-10.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics. |

**New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills**

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 9.4.12.CI.1 | Demonstrate the ability to reflect, analyze, and use creative skills and ideas. |
| 9.4.12.CT.2 | Explain the potential benefits of collaborating to enhance critical thinking and problem solving. |

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|---|---|
| 9.4.12.TL.1 | Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task. |
| 9.4.12.TL.3 | Analyze the effectiveness of the process and quality of collaborative environments. |
| 9.4.12.CI.1 | Demonstrate the ability to reflect, analyze, and use creative skills and ideas. |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |

**Instructional Focus**

**Unit Enduring Understandings**

- The foundation for the development of Euclidean geometry is a formal, rigorous study of mathematical relationships.
- Empirical verification (inductive reasoning) is an important part of the process of proving, but it can never, by itself, constitute a formal proof.
- Geometry uses a wide variety of kinds of proofs.
- We can communicate mathematical ideas effectively in a variety of modalities.

**Unit Essential Questions**

- What makes a statement true?
- What are the characteristics of a logical argument?
- Which is better: inductive or deductive reasoning?

**Content Understanding**

- There are two types of reasoning: inductive and deductive.
- There are rules and laws of logic that form the basis of a valid argument.
- Geometry is based on postulates that we accept as true without proof.

**Content Questions**

- How are the foundations of logical reasoning used to develop and improve conjectures?
- How does the application of logical reasoning facilitate understanding geometric relationships?
- How do lines, planes and pairs of angles relate?
- How can the facts of angle pairs be used to solve mathematical problems and prove geometric statements?
- How can lines and angles be used to study relationships within triangles and polygons?

**Objectives**

**Students will know:**

- **Vocabulary:** biconditional, converse, contrapositive, tautology, truth table, flow proof, Laws of Inference, inductive reasoning, deductive reasoning, parallel lines, parallel planes, transversal, alternate interior angles, alternate exterior angles, same side interior and exterior angles, corresponding angles, vertical angles, exterior angles of a triangle, complementary, supplementary, adjacent
- **Theorems:** theorems dealing with parallel lines, proving lines parallel and related theorems to parallel lines, Triangle Sum Theorem, Polygon Sum Theorem, Exterior Angle Theorem, Vertical Angle Theorem, Congruent Complements and Congruent Supplements Theorems, Right Angle Theorem

**Students will be able to:**

- Reason both inductively and deductively
- Express a statement in symbolic form
- Develop a truth table based on an argument
- Determine whether a statement is tautology through the development of a truth table
- Apply the Laws of Inference
- Analyze conditional statements and rewrite them in different forms
- Demonstrate elementary deductive reasoning
- Distinguish between intersecting lines and parallel lines

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|---|
| ● State and apply the postulates and theorems about parallel lines, planes, and pairs of angles.<br>● State and apply the theorems relating to the angle sum of a triangle and polygon. |
| **Evidence of Learning** |
| **Assessment** |
| Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data. |
| **Resources** |
| **Core Text:** Geometry by Big Ideas Learning - Larson and Boswell 2022<br>Suggested Resources: Trapeze Education |

# Unit 3: Triangles and Transformations

**Content Area:** Mathematics

**Course & Grade Level:** Geometry Honors and Accelerated/Ninth grade

**Summary and Rationale**

Having mastered the use of the tangent ratio to solve problems in a previous section, this unit introduces two new trigonometric ratios and their inverses- sine and cosine. This unit is a continuation of the development of the tangent concept with a focus on higher order thinking skills and applications.

Students will establish the foundation for figure congruence which relies on the definition of transformations. Students will build an understanding of rigid motions, including translations, reflections and rotations, in order to develop notions about what it means for two objects to be congruent. Rigid motions are at the foundation of the definition of congruence, and students should recognize that they preserve distance and angle measure. Dilations are at the foundation of the definition of similarity, and students should recognize that they preserve angle measure. Students will be encouraged to use a variety of tools, in order to transform given figures to prove congruence and similarity.

Students will use their knowledge of congruence to build an understanding of congruent triangles. Students will explore properties and relationships of sides and angles of triangles. They will apply the SSS, SAS, ASA, AAS and HL Triangle Congruence Postulates/Theorems to solve problems and prove various theorems about triangles throughout the unit, including those pertaining to congruence, mid-segments, perpendicular bisectors, angle bisectors, medians, altitudes, and special right triangles. Constructions can enhance understanding of geometric properties.

**Recommended Pacing**

36 days

**New Jersey Student Learning Standards for Mathematics**

| **Standard: Standards for Mathematical Practice** | |
|---|---|
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics. |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning. |
| **Standard:** G-CO.B Understand congruence in terms of rigid motions. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. |
| **Standard:** G-SRT.C Define trigonometric ratios and solve problems involving right triangles. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. |
| 7 | Explain and use the relationship between the sine and cosine of complementary angles. |

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|---|---|
| 8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. |
| **Standard:** G-SRT.D Apply trigonometry to general triangles. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 9 | Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. |
| 10 | Prove the Laws of Sines and Cosines and use them to solve problems. |
| 11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). |
| **New Jersey Student Learning Standards for English Language Arts Companion Standards** | |
| **Standard:** Science Key Ideas and Details | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| RST.9-10.3 | Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. |
| **Standard:** Science Craft and Structure | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| RST.9-10.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics. |
| **New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 9.4.12.CI.1 | Demonstrate the ability to reflect, analyze, and use creative skills and ideas. |
| 9.4.12.CT.2 | Explain the potential benefits of collaborating to enhance critical thinking and problem solving. |
| 9.4.12.TL.1 | Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task. |
| 9.4.12.TL.3 | Analyze the effectiveness of the process and quality of collaborative environments. |
| **New Jersey Student Learning Standards for Technology** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |
| **Instructional Focus** | |
| **Unit Enduring Understandings** | |
| ● We can communicate mathematical ideas effectively in a variety of modalities.<br>● Empirical verification and inductive reasoning are an important part of the process of proving, but can never constitute a formal proof.<br>● Making sense of others' arguments and determining their validity are proof-related activities.<br>● Trigonometry is the study of triangles – more specifically, the study of the angles and dimensions of triangles. Although this might sound simple, trigonometry is a vital part of modern engineering, design, architecture and other fields. | |
| **Unit Essential Questions** | |
| ● What is trigonometry?<br>● How are these abstract ratios used to make sense of the world around us? | |
| **Content Understanding** | |
| ● Extend algebraic skills to problem solving with geometric concepts. | |

- The processes of proving include a variety of activities, such as developing conjectures, considering the general case, exploring with examples, looking for structural similarities across cases, and searching for counterexamples.
- A proof can have many different valid representational forms, including narrative, two-column presentation, flow or algebraic form.
- Coordinate geometry can be used to represent and verify geometric/algebraic relationships.

**Content Questions**

- How are the sine, cosine and tangent related?
- How can right triangles be solved?
- How can trigonometry be applied to real world situations?
- How can we use geometric reasoning to prove statements about triangles?
- What is the criteria for triangles to be congruent/similar? How does this relate to transformations?
- What are the relationships between sides and angles of a triangle?
- What relationships exist between the trigonometric functions and the Pythagorean theorem?

**Objectives**

**Students will know:**

- **Terms:** reflection, rotation, translations, dilation, corresponding parts, isosceles triangle, equilateral triangle, parts of an isosceles and equilateral triangle, parts of a right triangle, median, altitude, midsegment, perpendicular bisector, angle bisector, sine, cosine, arcsine, arccosine, angle of depression, angle of elevation, oblique triangle, bearings, polygon, regular
- **Formulas:** Area of a triangle, Heron's Formula, Law of Sines, Law of Cosines
- **Theorems:** AAS, HL Theorem, Isosceles Triangle Theorem, Converse of Isosceles Triangle Theorem, Pythagorean Theorem, converse of the Pythagorean Theorem, theorems about the lengths of the sides of an acute/obtuse triangle, 45˚ - 45˚ - 90˚ Theorem, 30˚- 60˚ - 90˚ Theorem
- Concepts of rigid versus non-rigid motion
- Properties of transformations
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor

**Students will be able to:**

- Perform rigid motions to transform figures, and to predict the effect of a given rigid motion on given figures
- Use the definition in terms of rigid motions to decide if figures are congruent
- Identify the corresponding parts of congruent figures
- Apply SSS Theorem, the SAS Theorem, the ASA Theorem, and the AAS Theorem, HL Theorem to prove triangles congruent
- Deduce information about segments and angles after proving that two triangles are congruent.
- Apply the theorems about isosceles triangles
- State, apply, and prove the Pythagorean Theorem
- State and apply the converse of the Pythagorean Theorem and related theorems about obtuse and acute triangles
- Find the value of trigonometric functions of acute angles in a right triangle
- Solve and apply problems using right triangle trigonometry
- Prove Law of Sines and Law of Cosines
- Use Law of Sines and Law of Cosines to solve triangles
- Apply Law of Sines and Law of Cosines to real-world problems
- Determine when it is appropriate to use Law of Sines (AAS, ASA, SSA Triangles) and Law of Cosines (SAS, SSS Triangles)
- Derive formulas for the area of a triangle in the SAS (using the sine function) and SSS (Heron's formula)
- cases
- Find the area of a triangle in the SAS (using the sine function)
- Apply Law of Sines/Cosines to surveying and navigation problems

- Determine the lengths of two sides of a 45˚ - 45˚ - 90˚ or a 30˚ - 60˚ - 90˚ triangle when the length of the third side is known
- Relate sines and cosines of complementary angles
- Solve right triangle problems by using the sine, cosine and tangent ratios
- Solve applications using bearings, area of a triangle, Law of sines and cosines

**Evidence of Learning**

**Assessment**

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

**Resources**

**Core Text:** Geometry by Big Ideas Learning - Larson and Boswell 2022
Suggested Resources: Trapeze Education

# Unit 4: Quadrilaterals

**Content Area:** Mathematics

**Course & Grade Level:** Geometry Honors and Accelerated/Ninth grade

**Summary and Rationale**

In this unit, students will build on their knowledge of quadrilaterals from middle school. Students will re-explore properties and conditions of special quadrilaterals such as parallelograms, rhombuses, rectangles, squares, kites and trapezoids. They will combine this previous knowledge with their new knowledge of trigonometric ratios to deductively prove theorems and solve complex problems involving quadrilaterals. Additionally, they will revisit coordinate proofs using distance and slope involving different types of quadrilaterals.

**Recommended Pacing**

9 days

**New Jersey Student Learning Standards for Mathematics**

| **Standard: Standards for Mathematical Practice** | |
|---|---|
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics. |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning. |
| **Standard:** G-CO.C Prove geometric theorems | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 11 | Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. |
| **Standard:** G-GPE.B Use coordinates to prove simple geometric theorems algebraically. | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 4 | Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2). |

**New Jersey Student Learning Standards for English Language Arts Companion Standards**

| **Standard:** Science Key Ideas and Details | |
|---|---|
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| RST.9-10.3 | Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. |
| **Standard:** Science Craft and Structure | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| RST.9-10.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics. |

| New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills | |
|---|---|
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 9.4.12.CI.1 | Demonstrate the ability to reflect, analyze, and use creative skills and ideas. |
| 9.4.12.CT.2 | Explain the potential benefits of collaborating to enhance critical thinking and problem solving. |
| 9.4.12.TL.1 | Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task. |
| 9.4.12.TL.3 | Analyze the effectiveness of the process and quality of collaborative environments. |
| **New Jersey Student Learning Standards for Technology** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |

**Instructional Focus**

**Unit Enduring Understandings**

- Euclidean geometry is a system of postulates and theorems.
- We can communicate mathematical ideas effectively in a variety of modalities.
- Hierarchies of properties occur often in mathematics. Understanding a hierarchy develops our reasoning ability.
- Any four non-collinear points in space form a quadrilateral. There are many quadrilaterals in the world around us.

**Unit Essential Questions**

- How can we use the general and specific properties of quadrilaterals to reason effectively?
- Why study quadrilaterals?
- How do geometric constructions enhance understanding of the geometric properties of objects?

**Content Understanding**

- Quadrilaterals have a hierarchy of properties.
- The processes of proving include a variety of activities, such as developing conjectures, considering the general case, exploring with examples, looking for structural similarities across cases, and searching for counterexamples.
- Making sense of others' arguments and determining their validity are proof-related activities.
- A proof can have many different valid representational forms, including narrative, two-column presentation, flow or algebraic form.

**Content Questions**

- How can the properties of geometric figures be verified using a coordinate plane or formal proof?
- What minimum conditions are necessary to prove a quadrilateral is a special quadrilateral?

**Objectives**

**Students will know:**

- **Terms**: parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid, kite, midpoint quadrilateral, median of a trapezoid
- **Theorems**: Theorems about the properties of a parallelogram, theorems about the ways to prove a quadrilateral is a parallelogram, theorems about the properties of a rectangle and rhombus, theorems about the ways to prove a parallelogram is a rectangle or rhombus, theorems about the properties of a trapezoid/isosceles trapezoid, theorems about properties of kites

**Students will be able to:**

- Apply the definition of a parallelogram and the theorems about the properties of a parallelogram
- Prove using coordinate geometry and formal proof that quadrilaterals are special quadrilaterals
- Apply the definitions then identify and prove the special properties of a rectangle, a rhombus, a square, a kite, a trapezoid, and an isosceles trapezoid

| Evidence of Learning |
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| **Assessment** |
| Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data. |
| **Resources** |
| **Core Text:** Geometry by Big Ideas Learning - Larson and Boswell 2022<br>Suggested Resources: Trapeze Education |

# Unit 5: Circles

**Content Area:** Mathematics

**Course & Grade Level:** Geometry Honors and Accelerated/Ninth grade

**Summary and Rationale**

The purpose of this unit is to have students experience mathematics as a language for understanding and explaining phenomena; not just a way to solve specific problems. Students will build on their understanding of congruence and similarity to investigate relationships between circles. In addition, students will explore and prove relationships between parts of circles. Students should understand how these parts relate to segment lengths and angle measures, and how this relates back to similarity. Students will use formulas for circumference and area to explore and derive the formula for arc length and area of a sector then apply this knowledge to find the area of a segment of a circle. Students will use the knowledge from this section in conjunction with quadrilaterals and other polygons to problem solve for missing dimensions and areas.

**Recommended Pacing**

13 days

**New Jersey Student Learning Standards for Mathematics**

**Standard: Standards for Mathematical Practice**

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics. |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning. |

**Standard:** G-C.A Understand and apply theorems about circles.

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 1 | Prove that all circles are similar. |
| 2 | Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. |
| 3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. |

**Standard:** G-C.B Find arc lengths and areas of sectors of circles.

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. |

**New Jersey Student Learning Standards for English Language Arts Companion Standards**

**Standard:** Science Key Ideas and Details

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|

| | |
|---|---|
| RST.9-10.3 | Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. |
| **Standard:** Science Craft and Structure | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| RST.9-10.4 | Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics. |
| **New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 9.4.12.CI.1 | Demonstrate the ability to reflect, analyze, and use creative skills and ideas. |
| 9.4.12.CT.2 | Explain the potential benefits of collaborating to enhance critical thinking and problem solving. |
| 9.4.12.TL.1 | Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task. |
| 9.4.12.TL.3 | Analyze the effectiveness of the process and quality of collaborative environments. |
| **New Jersey Student Learning Standards for Technology** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |
| **Instructional Focus** | |
| **Unit Enduring Understandings** | |
| ● We can communicate mathematical ideas effectively in a variety of modalities.<br>● Pi is a ratio used to determine area and circumference.<br>● The study of circles helps us to understand the properties of our spherical planet.<br>● Because of its unique properties, a circle is considered by some to be a perfect figure. | |
| **Unit Essential Questions** | |
| ● Can we ever know the area or circumference of a circle exactly?<br>● What makes a circle perfect?<br>● How are circles used to understand our planet? | |
| **Content Understanding** | |
| ● The processes of proving include a variety of activities, such as developing conjectures, considering the general case, exploring with examples, looking for structural similarities across cases, and searching for counterexamples.<br>● Making sense of others' arguments and determining their validity are proof-related activities.<br>● A proof can have many different valid representational forms, including narrative, two-column presentation, flow or algebraic form.<br>● We use similarity and congruence to investigate and justify the relationships between circles.<br>● Changing the diameter or radius will affect the area and circumference of a circle differently. | |
| **Content Questions** | |
| ● How are segments within circles, such as radii, diameters, and chords, related to each other?<br>● What is the relationship of their measurements?<br>● How do inscribed, circumscribed, and central angles relate to each other?<br>● How can various figures be inscribed in a circle using various tools?<br>● How do the properties of these figures relate to the parts of a circle?<br>● What relationships exist between segments and angles formed by tangents, secants and chords?<br>● How are circular arcs and area of sectors/segments calculated? | |

**Objectives**

**Students will know:**

- **Terms:** circle, center, radius, chord, diameter, secant, tangent, point of tangency, congruent circles, concentric circles, inscribed and circumscribed circles, great circle, common tangents, tangent circles, central angle, arc, minor and major arc, semicircle, measure of an arc, adjacent arcs, congruent arcs, inscribed angle, intercepted arc, arc length, sector, segment
- **Postulate:** Arc Addition Postulate
- **Theorems:** Theorems dealing with tangents and chords (and their converses), same circles and congruent circles, inscribed angles, tangent-tangent angles, tangent-chord angles, chord-chord angles, tangent-secant angles. Theorems dealing with segments of a circle including chord-chord segments, tangent-secant segments, secant-secant segments, Theorems dealing with tangents and inscribed angles

**Students will be able to:**

- Define a circle, a sphere and terms related to them
- Prove that all circles are similar
- Identify and describe relationships among inscribed angles, radii, and chords
- Recognize inscribed (circumscribed) polygons and circumscribed (inscribed) circles
- Construct the inscribed and circumscribed circles of a triangle, and prove the properties of angles for a quadrilateral inscribed in a circle
- Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality
- Derive the formula for the area of a sector
- Apply theorems about chords of a circle
- Solve problems and prove statements involving inscribed angles and angles formed by chords, secants and tangents of a circle
- Solve problems involving lengths of chords, secant segments and tangent segment

**Evidence of Learning**

**Assessment**

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

**Resources**

**Core Text:** Geometry by Big Ideas Learning - Larson and Boswell 2022
Suggested Resources: Trapeze Education

# Unit 6: Surface Area and Volume

**Content Area:** Mathematics

**Course & Grade Level:** Geometry Honors and Accelerated/Ninth grade

**Summary and Rationale**

Surface area and volume problems offer great opportunities for students to explore all the skills they have learned in the course. Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of area, and volume formulas. Students will model problems with three dimensional figures and will consider the shapes of the two-dimensional cross-sections of those figures. In addition, students will consider the figures created by the rotation of two-dimensional figures about a line.

**Recommended Pacing**

8 days

**New Jersey Student Learning Standards for Mathematics**

**Standard: Standards for Mathematical Practice**

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 1 | Make sense of problems and persevere in solving them. |
| 2 | Reason abstractly and quantitatively. |
| 3 | Construct viable arguments and critique the reasoning of others. |
| 4 | Model with mathematics. |
| 5 | Use appropriate tools strategically. |
| 6 | Attend to precision. |
| 7 | Look for and make use of structure. |
| 8 | Look for and express regularity in repeated reasoning. |

**Standard:** G-GMD.A Explain volume formulas and use them to solve problems.

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. |
| 3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. |

**Standard:** G-GMD.B Visualize relationships between two-dimensional and three-dimensional objects.

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. |

**Standard:** G-MG.A Apply geometric concepts in modeling situations.

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| 1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). |
| 2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). |
| 3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). |

**New Jersey Student Learning Standards for English Language Arts Companion Standards**

**Standard:** Science Key Ideas and Details

| CPI # | Cumulative Progress Indicator (CPI) |
|---|---|
| RST.9-10.3 | Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. |
| **New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 9.4.12.CI.1 | Demonstrate the ability to reflect, analyze, and use creative skills and ideas. |
| 9.4.12.CT.2 | Explain the potential benefits of collaborating to enhance critical thinking and problem solving. |
| 9.4.12.TL.1 | Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task. |
| 9.4.12.TL.3 | Analyze the effectiveness of the process and quality of collaborative environments. |
| **New Jersey Student Learning Standards for Technology** | |
| **CPI #** | **Cumulative Progress Indicator (CPI)** |
| 8.1 | All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge. |
| **Interdisciplinary Standards: NGSS** | |
| HS.ESS2 | Earth's Systems<br>Science example: When coastal erosion is measured, what are its units? What does this tell you about what quantity is being measured? (p.33 NGSS Crosswalk) |

**Instructional Focus**

**Unit Enduring Understandings**

- We can communicate mathematical ideas effectively in a variety of modalities.
- Surface area and volume help quantify the world around us.

**Unit Essential Questions**

- How do you describe the size of a three-dimensional figure?

**Content Understanding**

- The process of calculating surface area and volume of 3-dimensional solids includes a variety of dimensions, such as height, slant height, radius, base edge length, and so forth.

**Content Questions**

- How can the formulas for area and volume be explained using various tools and visual or tactile representations?
- How can surface area and volume be used to solve real world applications of 3D solids?
- How can the formulas for surface area of volume of 3D figures help calculate the area of volume of composite solids?
- What will be the three-dimensional result of rotating a two-dimensional figure about a line?

**Objectives**

**Students will know:**

- **Terms:** prism, base, altitude, lateral face, lateral edge, right prism, oblique prism, lateral area, surface area, cube, volume, regular pyramid, vertex, slant height, cylinder, cone, sphere, similar solids
- **Theorems:** Theorems dealing with lateral area, total area and volume of prisms, pyramids, cylinders and cones, theorems with area and volume of spheres, theorems involving ratios of two similar solids

**Students will be able to:**

- Identify the parts of prisms, pyramids, cones, cylinders, and spheres
- Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects
- Find the lateral areas, total areas and volumes of right prisms, regular pyramids, right cylinders, right cones

- Find the area and volume of a sphere
- State and apply the properties of similar solids

**Evidence of Learning**

**Assessment**

Assessment plan may include teacher designed formative and summative assessments, a district common assessment, analysis of PSAT and NJSLA data.

**Resources**

**Core Text:** Geometry by Big Ideas Learning - Larson and Boswell 2022
Suggested Resources: Trapeze Education