West Windsor-Plainsboro Regional School District
Algebra & Trigonometry
Summer 2023

Math Equity Statement
ALL learners should have access to rigorous, high-level mathematical content in an environment where risk-taking, deep conceptual understanding, and growth mindset are the norm.

Core Beliefs:
Our district's strategic goals prioritize teaching and learning from a productive standpoint. Building upon the principles of *Catalyzing Change in High School Mathematics*, we aim to cultivate equitable mathematics practices and shift from deficit-based to productive beliefs. According to the National Council of Teachers of Mathematics (NCTM, 2020), "Mathematics education must be equitable, ensuring that each and every student has access to high-quality mathematics teaching and learning opportunities." Our objective is for every student to perceive themselves as capable, knowledgeable, and meaning-makers in mathematics.

Drawing from *Catalyzing Change* and *Mathematical Mindsets* by Jo Boaler (2016), we embrace the following beliefs:

- All students are capable of learning mathematics at high levels.
- All students will progress on their mathematical journey.
- Developing a growth mindset is essential for learning.
- Visual and deep thinking enhance mathematical understanding.
- Mathematics learning is fostered through discourse and collaboration.
- Mistakes are integral to the learning process.

Math Workshop:
Catalyzing Change states that teaching should provide opportunities for each and every student to develop a positive mathematical identity, recognizing their own mathematical abilities and potential. The Math Workshop instructional model enables meaningful mathematics engagement, reflection, and the realization of students' potential as mathematicians. By incorporating student choice, problem-solving, targeted small group instruction, and deliberate practice of critical grade-level concepts (Lempp, 2017), Math Workshop creates an environment where students feel comfortable taking intellectual risks. Sienna (2009) outlines four values to support students in taking risks and fostering discourse, which include:

- Valuing the thinking process alongside correct answers.
- Valuing problems that allow for multiple solutions.
- Valuing inquisitive responses.
- Valuing tolerance for mistakes (Sienna, 2009, p. 68).

Math Workshop embraces these values and fosters a supportive, collaborative learning environment for all students. It is the instructional model employed by our dedicated teachers.

Unit 1: Algebra Essentials	
Content Area: Mathematics	
Course & Grade Level: 11-12	
Summary and Rationale	
Learning the essentials of algebra is of paramount importance as it serves as a foundational tool in numerous aspects of life. Algebraic skills not only enhance problem-solving abilities but also promote critical thinking and logical reasoning. Whether in the field of science, technology, engineering, or mathematics, algebra provides the framework for understanding complex relationships, analyzing data, and making informed decisions. Moreover, it is a gateway to higher-level mathematics and various other disciplines, empowering individuals to excel in academics and professional pursuits. Beyond formal education, algebra equips individuals with practical skills, enabling them to manage finances, interpret graphs, and solve everyday challenges. Overall, mastering the essentials of algebra equips individuals with a powerful mental toolkit, fostering intellectual growth and equipping them for success in an increasingly complex and interconnected world.	
Recommended Pacing	
41 days	
New Jersey Student Learning Standards for Mathematics	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: A.APR.A: Perform arithmetic operations on polynomials	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Standard: A.APR.B: Understand the relationship between zeros and factors of polynomials	
CPI #	**Cumulative Progress Indicator (CPI)**
2	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Standard: A.CED.A: Create equations that describe numbers or relationships	
CPI #	**Cumulative Progress Indicator (CPI)**
2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Standard: A.CED. B: Solve equations and inequalities in one variable	
CPI #	**Cumulative Progress Indicator (CPI)**
3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4a	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form.
4b	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
Standard: A.CED.C: Solve systems of equations	
CPI #	**Cumulative Progress Indicator (CPI)**
5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.
Standard: F.IF.B: Interpret functions that arise in applications in terms of the context	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
Standard: F.IF.C: Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	

CPI #	Cumulative Progress Indicator (CPI)
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience
Interdisciplinary Standards (fill-in Science, or SS, or Math, etc..)	
CPI #	**Cumulative Progress Indicator (CPI)**
HS.PS2	Motion and Instability: Forces and Interactions
Science examples: (1) Informally fit a quadratic function to the position-time data for a cart that rolls up an incline (slowing as it climbs, then reversing direction and speeding up as it descends). Use the algebraic expression for the fitted function to determine the magnitude of the cart's acceleration and initial speed. Over several trials, graph various quantities (such as acceleration vs. angle, or peak displacement vs. initial speed squared), and interpret the results. (2) Calculate and interpret the average speed of a moving object by using data from a distance-time graph. - NGSS, Appendix L, page 29	

Instructional Focus

Unit Enduring Understandings

- Functions are a special type of relationship or rule that uniquely associates members of one set with members of another set. Their characteristics impact their behavior and representation.
- Many real world problems can be solved through the use of algebraic representation and reasoning.
- Graphs and equations of functions are alternative (and often equivalent) ways for depicting and analyzing patterns and identifying solutions.
- The rules of arithmetic and algebra can be used together, with the concept of equivalence, to transform expressions, and equations so solutions can be found to solve problems.

Unit Essential Questions

- How can mathematical models be used as tools to best describe change and help represent real-life situations?
- How do the components of function affect the characteristics of its graph? How are factors, zeros and x-intercepts of a function related?
- How do functions model real-world problems and their solutions?
- How do you determine the most efficient and appropriate method for solving problems?

Content Understandings

- A function is a relation where every input is paired with exactly one output.
- A polynomial function has distinguishing behaviors. Its algebraic form gives information about its graph and its graph gives information about its algebraic form.
- Functions can be evaluated to help create multiple representations and analyze patterns.
- Domain of a function is a set of input values, and the range is the set of output values.
- Linear functions have a constant rate of change and create a line when graphed.
- A solution of a system of linear equations in two variables can be an ordered pair (point of intersection), no solution, or infinitely many solutions.
- There are different methods to solve systems.
- Quadratic equations and functions can be solved using a variety of methods

- A polynomial is a monomial or a sum of monomials; each monomial is a numeral, variable or a product of variables. Thus the form of a polynomial function is $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... a_1 x + a_0$ where each power of *x* is a whole number and each coefficient is real.
- Polynomial functions as graphs must be continuous (no breaks, gaps or holes). Their domain is always "all real numbers". Their turns are always rounded.
- For polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... a_1 x + a_0$, the end behavior is determined by the sign of a_n and whether *n* is even or odd. There are four possible end behaviors to consider.
- Identifying extrema (mins/maxes) and increasing/decreasing sections of a polynomial function's graph help us to describe its distinguishing characteristics.
- You can divide polynomials using an algorithm that is similar to the long division algorithm that you use to divide whole numbers.

Content Questions

- What are the possible relationships that can occur with simultaneous equations?
- What is a reasonable domain and range for a given function?
- Which is the most efficient method for solving a quadratic equation?
- What do the solutions of a quadratic equation tell you about its graph?
- How can you determine the number of solutions of a system of equations?
- How does polynomial long division resemble numerical long division? How can this connection be used to understand the Division Algorithm?
- What is the difference between turning points and inflection points?

Objectives

We are learning to/that:

- Identify functions and their characteristics such as domain and range, increasing and decreasing, and critical points
- Use function notation when performing function operations and compositions
- Graph various forms of linear functions.
- Write linear functions using points, slopes, and parallel & perpendicular relationships.
- Identify and graph piecewise functions, including absolute value functions
- Solve systems of equations and real world problems involving three variables
- Factor quadratics and polynomial expressions
- Identify characteristics of quadratic functions like axis of symmetry, increasing, and decreasing.
- Choose and use the best method of solution for a quadratic equation
- Add, subtract, multiply,and divide polynomials (synthetic and long)
- Identify characteristics of quadratic functions like axis of symmetry, increasing, and decreasing
- Graph polynomials and write equations of polynomials.
- Use the factor theorem, the remainder theorem, and the rational roots theorem

Evidence of Learning

☑ Formative Assessment

☑ Summative Assessment

☑ Alternative Assessment

☑ Benchmark

Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. [Accommodations]

Resources
Core Text: Integrated III, Big Ideas, Larson, Boswell, 2016

Unit 2: Advanced Topics in Algebra	
Content Area: Mathematics	
Course & Grade Level: 11-12	
Summary and Rationale	
Exponential and logarithmic functions play a crucial role in various fields of science, engineering, economics, and technology due to their unique mathematical properties and widespread applications. Exponential functions represent rapid growth or decay phenomena, making them essential in modeling population growth, compound interest, radioactive decay, and the spread of diseases. They are also fundamental in describing natural processes such as bacterial growth, chemical reactions, and environmental changes. On the other hand, logarithmic functions are the inverse of exponential functions and find significant application in data analysis, signal processing, and algorithm complexity analysis. They are particularly valuable in converting multiplicative relationships into additive ones, simplifying complex calculations and enabling more straightforward data visualization. Whether it's predicting trends, solving differential equations, designing algorithms, or understanding various phenomena, exponential and logarithmic functions provide powerful tools that are indispensable in modern scientific and technological advancements.	
Recommended Pacing	
19 days	
New Jersey Student Learning Standards for Mathematics	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: A.REI.A: Understand solving equations as a process of reasoning and explain the reasoning	
CPI #	**Cumulative Progress Indicator (CPI)**
2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Standard: F.IF.C: Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8b	Use the properties of exponents to interpret expressions for exponential functions.
Standard: F.BF.B: Build new functions from existing functions	
CPI #	**Cumulative Progress Indicator (CPI)**
5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
Standard: F.LE.A: Construct and compare linear and exponential models and solve problems	

CPI #	Cumulative Progress Indicator (CPI)
1c	Distinguish between situations that can be modeled with linear functions and with exponential functions. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab^ct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience
Interdisciplinary Standards Science	
HS.LS1	From Molecules to Organisms: Structures and Processes
Science example: Use a spreadsheet or other technology to simulate the doubling in a process of cell division; graph the results; write an expression to represent the number of cells after a division in terms of the number of cells beforehand; express this in closed form as a population size in terms of time. Discuss real-world factors in the situation that lead to deviation from the exponential model over time. - NGSS, Appendix L, page 30	
Instructional Focus	
Unit Enduring Understandings	
• Mathematicians use algebraic and graphical representations to generalize patterns and relationships • Inverse operations allow us to state mathematical facts in a variety of ways. • Mathematicians can use a variety of equations to model, interpret and explain real life phenomena.	
Unit Essential Questions	
• What is the relationship between logarithmic and exponential functions? • How do functions model real-world problems and their solutions? • How do you determine the most efficient and appropriate method for solving problems?	
Content Understandings	
• Rational equations are used to model, understand and explain real life situations • Domain restrictions affect solutions of an algebraic equation	

- Mathematicians use exponential and logarithmic equations to model, interpret and explain real life phenomena
- Fluency between radical and rational exponential form allows us to simplify efficiently
- The inverse relationship between exponential and logarithmic functions can be used to graph logarithmic functions
- Logarithms and exponents have corresponding properties

Content Questions

- What is a logarithm and how do mathematicians use them?
- How can you use properties of exponents to derive properties of logarithms?
- What are some of the characteristics of the graph of a logarithmic function?
- How do we solve real world application problems involving exponential growth and decay?

Objectives

We are learning to/that:

- Add/subtract/multiply and divide rational expressions
- Solve rational equations
- Simplify expressions in radical and rational exponent form using properties of exponents
- Find nth roots
- Evaluate expressions containing exponents and logarithms using the properties of exponents
- Solving equations involving nth roots
- Graphing exponential functions
- Applications of exponential growth and decay
- Using inverses to graph logarithmic functions
- Use the properties of logarithms to evaluate, expand and condense
- Use the change of base formula to evaluate logs
- Rewrite logarithms as exponentials and vice versa
- Solve exponential and logarithmic equations using various methods

Evidence of Learning

- ☑ Formative Assessment
- ☑ Summative Assessment
- ☑ Alternative Assessment
- ☑ Benchmark

Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations

Resources

Core Text: Integrated III, Big Ideas, Larson, Boswell, 2016

Unit 3: Transformations	
Content Area: Mathematics	
Course & Grade Level: 11-12	
Summary and Rationale	
Teaching transforming graphs in math class serves multiple important purposes. First and foremost, it helps students grasp a deeper understanding of functions and their behavior. By exploring how various transformations such as translations, reflections, stretches, and compressions affect the shape and position of graphs, students gain valuable insights into the underlying patterns and relationships between different functions. These skills are crucial for advanced math and science courses, as well as for solving real-world problems that involve data analysis and modeling. Additionally, studying transforming graphs nurtures students' spatial reasoning and visualization abilities, as they learn to mentally manipulate and predict changes in graphs without relying solely on equations. Moreover, it cultivates critical thinking and problem-solving skills, as students are challenged to interpret and apply transformations in various contexts. Ultimately, understanding transforming graphs equips students with a powerful toolkit for interpreting and communicating complex mathematical concepts, laying the groundwork for their future success in STEM fields and beyond.	
Recommended Pacing	
11 days	
New Jersey Student Learning Standards for Mathematics	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: F.IF.B: Interpret functions that arise in applications in terms of the context	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Standard: F.BF.B: Build new functions from existing functions	
CPI #	**Cumulative Progress Indicator (CPI)**
3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and

	illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4a	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠1.
4b	Verify by composition that one function is the inverse of another.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience
Instructional Focus	
Unit Enduring Understandings	
• The relationships between functions and their inverses relate to their domain and range • Patterns and relationships can be represented graphically, numerically, symbolically, and verbally. Each representation has its advantages and disadvantages. • Mathematicians can use a variety of equations to model, interpret and explain real life phenomena. • Functions model real world situations using various equations, graphs and tables. These models help mathematicians understand and explain real life data. • There are sets of functions, called families, in which each function is a transformation of a specific function called the parent.	
Unit Essential Questions	
• Why are inverse functions important and how are they used to solve problems? • How are the function families similar and how are they different?	
Content Understandings	
• Restrictions in the domain of a rational function result in discontinuities • Domain restrictions affect solutions of algebraic equations. • The vertical asymptotes of a rational function occur at the zeros of the polynomial in the denominator, as long as those zeros are not zeros of the polynomial in the numerator. • A horizontal asymptote models a constant end behavior of a rational function • Extraneous solutions occur when eliminating the common denominator, thus changing the set of possible solutions. • Transformations are changes to the parent function(s) • The relationships between functions and their inverses relate to their domain and range	
Content Questions	

- How are the techniques for graphing and transforming functions similar and different when comparing different function families?
- What is the importance of an inverse function?
- What do asymptotes of functions represent?
- When do we need to determine the domain and range of functions?
- How do we write equations of functions from verbal descriptions?

Objectives

We are learning to/that:

- Perform transformations of linear, quadratic, cubic, absolute value, radical and exponential functions
- Graph transformations of linear, quadratic, cubic, absolute value, radical and exponential functions
- Write equations of graphs
- Determine the domain and range of functions
- Graph functions and their inverses
- Determine whether a function is continuous or discontinuous
- Identify discontinuity
- Construct and graph functions with discontinuity
- Graph rational functions
- Determine vertical, horizontal and slant asymptotes

Evidence of Learning

- ☑ Formative Assessment
- ☑ Summative Assessment
- ☑ Alternative Assessment
- ☑ Benchmark

Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations

Resources

Core Text: Integrated III, Big Ideas, Larson, Boswell, 2016

Unit 4: Trigonometric Ratios and Functions	
Content Area: Mathematics	
Course & Grade Level: 11-12	
Summary and Rationale	
Studying trigonometric functions in an algebra class is essential because these functions establish a fundamental link between algebra and geometry, paving the way for a deeper understanding of both disciplines. Trigonometry plays a crucial role in solving problems involving angles, distances, and relationships between shapes, making it indispensable in various real-world applications such as engineering, physics, and architecture. By exploring trigonometric identities, students develop their algebraic manipulation skills while learning to simplify complex expressions and solve trigonometric equations. Furthermore, understanding trigonometric functions enables students to analyze periodic phenomena, comprehend waveforms, and model cyclic behavior in fields like physics and signal processing. Overall, the study of trig functions enriches students' mathematical toolkit, fostering a more comprehensive grasp of mathematical concepts and enhancing their problem-solving abilities across diverse disciplines.	
Recommended Pacing	
17 days	
New Jersey Student Learning Standards for Mathematics	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: F.TF.A: Extend the domain of trigonometric functions using the unit circle	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
Standard: G.SRT.C: Define trigonometric ratios and solve problems involving right triangles	
CPI #	**Cumulative Progress Indicator (CPI)**
6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7	Explain and use the relationship between the sine and cosine of complementary angles.
8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Standard: G.SRT.D: Apply trigonometry to general triangles	
CPI #	**Cumulative Progress Indicator (CPI)**

9	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10	Prove the Laws of Sines and Cosines and use them to solve problems.
11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience

Instructional Focus

Unit Enduring Understandings

- Trigonometry is a discipline that is based on the study of triangles.
- We use the trigonometric ratios and inverses to model phenomena and solve real-world problems.
- There are many instances of periodic data in the world around us and trigonometric functions can be used to model real world data that is periodic in nature.
- A problem solver understands what has been done, knows why the process was appropriate, and can support it with reasons and evidence.
- The calculator and other technologies are tools to supplement and clarify mathematical thinking, and answers from the calculator need to be anticipated and interpreted appropriately.

Unit Essential Questions

- What is the best way to measure an angle?
- How to use trigonometric ratios and its inverses to solve equations and real-world problems?

Content Understandings

- Trigonometric functions are ratios of sides of right triangles.
- Radian measure is a different unit system for measuring angles. Degree and radian measurement of an angle are used in different contexts of trigonometry.
- The relationship between sides and angles in a right triangle can be used to evaluate the six trigonometric functions, sine, cosine, tangent, cosecant, secant and cotangent of acute angles.
- Trigonometric functions can be used to solve real life problems involving indirect measurement.
- The Laws of sines and cosines can be used together to solve problems involving triangles.
- The Cartesian system considers counterclockwise as positive orientation.

Content Questions

- How can special right triangles help us find the coordinates of certain angles on the unit circle?
- How does symmetry help us extend our knowledge of the unit circle to an infinite number of angles?

- What is the difference between degree and radian measurement?
- How are the six trigonometric functions related?
- How to solve and apply problems using right triangle trigonometry?
- How to apply Law of Sines and Law of Cosines to real-world problems?

Objectives

We are learning to/that:

- Find the length of sides and measure of angles in a triangle by using right triangle trigonometric functions.
- Solve and apply problems using right triangle trigonometry.
- Understand the connection between the 6 trigonometric functions and their inverses
- Recognize and understand the difference between degree and radian measure when measuring angles.
- Convert between degree measure and radian measure, and use radian measure to find arc lengths and areas of sectors.
- Draw angles in standard position in the coordinate plane.
- Find coterminal angles of given angle by adding or subtracting 360° or 2π radians.
- Determine exact values of trigonometric functions of unit circle angles.
- Evaluate trigonometric functions of any angle, and use the unit circle to find trigonometric functions of quadrantal angles.
- Find and use reference angles to evaluate trigonometric functions.
- Evaluate the exact value of inverse sine, cosine, tangent functions.
- Use a calculator to evaluate inverse sine, cosine, tangent functions.
- Use the Law of Sines and the Law of Cosines to solve triangles.
- Apply Law of Sines and Law of Cosines to real-world problems
- Find areas of triangles using the sine function or when all three side lengths are known.

Evidence of Learning

☑ Formative Assessment

☑ Summative Assessment

☑ Alternative Assessment

☑ Benchmark

Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. [Accommodations]

Resources

Core Text: Integrated III, Big Ideas, Larson, Boswell, 2016

Unit 5: Graphing Sinusoidal Functions and Trigonometric Identities	
Content Area: Mathematics	
Course & Grade Level: 11-12	
Summary and Rationale	
Graphing trigonometric functions is of utmost importance as it provides a visual representation of the periodic behavior of these functions, enabling a deeper comprehension of their properties and applications. By plotting trig functions such as sine, cosine, and tangent, students gain insights into their oscillatory nature and observe how changing parameters affect their shapes and positions on the coordinate plane. This understanding proves invaluable when analyzing waveforms in physics, electrical engineering, and signal processing. Additionally, graphing trig functions enhances problem-solving skills as it allows students to visualize and interpret real-world scenarios involving periodic phenomena, such as sound waves, pendulum motion, and electromagnetic oscillations. Furthermore, comprehending the graphs of trig functions helps in solving trigonometric equations and establishing connections between algebraic manipulations and geometric concepts. Overall, graphing trig functions equips students with a powerful tool to interpret and analyze a wide range of phenomena, making it an indispensable skill in various STEM disciplines and beyond.	
Recommended Pacing	
15 days	
New Jersey Student Learning Standards for Mathematics	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: F.TF.B: Model periodic phenomena with trigonometric functions	
CPI #	**Cumulative Progress Indicator (CPI)**
5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Standard: F.TF.C: Prove and apply trigonometric identities	
CPI #	**Cumulative Progress Indicator (CPI)**
8	Prove the Pythagorean identity sin^2(θ) + cos^2(θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
New Jersey Student Learning Standards for English Language Arts **Companion Standards**	

Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience
Instructional Focus	

Unit Enduring Understandings

- A function that repeats its values in regular intervals is known as a periodic function. Trigonometric functions describe periodic phenomena.
- The graphs of sine and cosine model real-world phenomena.
- There are different ways to manipulate and simplify expressions to create equivalent expressions.
- There can be different strategies to solve a problem, but some are more effective and efficient than others are.

Unit Essential Questions

- How does the periodic nature of the trigonometric functions help to solve problems?
- How do you use the fundamental trigonometric identities to rewrite expressions?

Content Understandings

- Trigonometric graphs can offer a Cartesian representation of the periodic nature generated from unit circle values.
- Five key points can be used to graph a sinusoidal function.
- The basic form of a sinusoidal function can be written as y = A sin (B(x - C)) + D, where A is the amplitude or height of our function, B is the change in period defined by 2pi/B, C the horizontal shift, and D the vertical shift.
- Transformation of the basic sine and cosine functions can change the characteristics of its graphs in terms of shifts in amplitude, period, horizontal shift, vertical shift, and range.
- A trigonometric identity is a trigonometric equation that is true for every possible value of the input variable on which it is defined.
- Various strategies can be used to prove identities, including Pythagorean, reciprocal and quotient identities.
- Trigonometric Identities can be used along with algebraic methods to solve trigonometric equations.

Content Questions

- What are the differences/similarities among the behaviors of the sine and cosine functions?
- How does the unit circle relate to the graphs of sine and cosine functions?
- How do we interpret the key components (i.e. amplitude, period, phase shift, vertical shift, domain and range) of the graph of a sine and cosine function?
- How does transformation of the basic sine and cosine function affect the amplitude, period, horizontal shift, vertical shift, domain and range?
- How can two (or more) trigonometric expressions be verified as equivalent?

- What is an identity?
- What algebraic techniques and trigonometric properties are useful in verifying trigonometric identities and solving trigonometric equations?

Objectives

- Explore domains, ranges, amplitudes, and periods of sine and cosine functions.
- Stretch and Shrink graphs of sine and cosine functions vertically and horizontally.
- Translate graphs of sine and cosine functions vertically and horizontally
- Reflect graphs of sine and cosine functions in the x-axis.
- Write the equation of sine and cosine functions as $y = a\sin b(x - c) + d$ and $y = a\cos b(x - c) + d$
- Use trigonometric identities involving sine and cosine to find trigonometric values.
- Use trigonometric identities to evaluate and simplify trigonometric expressions and to solve trigonometric equations.
- Verify trigonometric identities.

Evidence of Learning

- ☑ Formative Assessment
- ☑ Summative Assessment
- ☑ Alternative Assessment
- ☑ Benchmark

Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations

Resources

Core Text: Integrated III, Big Ideas, Larson, Boswell, 2016

Unit 6: Matrices	
Content Area: Mathematics	
Course & Grade Level: 11-12	
Summary and Rationale	
Learning about matrices in Algebra class is highly significant as it opens the door to a wealth of applications in mathematics, science, engineering, and beyond. Matrices provide a powerful tool for representing and solving systems of linear equations, which are prevalent in various real-world problems such as network analysis, optimization, and data manipulation. Understanding matrix operations, such as addition, multiplication, and inversion, facilitates the manipulation and transformation of data, enabling students to analyze complex datasets and model real-life scenarios. Moreover, matrices are essential in fields like computer graphics, quantum mechanics, and economics, where they aid in solving problems involving transformations, quantum states, and input-output relationships. Learning about matrices cultivates critical thinking, logical reasoning, and problem-solving skills, as students explore the elegant structure and properties of these mathematical objects. Overall, mastering matrices equips students with a versatile mathematical toolset that underpins diverse disciplines and prepares them for future challenges in academic pursuits and professional careers.	
Recommended Pacing	
7 days	
New Jersey Student Learning Standards for Mathematics	
Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: N.VM.C: Perform operations on matrices and use matrices in applications.	
CPI #	**Cumulative Progress Indicator (CPI)**
6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8	Add, subtract, and multiply matrices of appropriate dimensions.
9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12	Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
New Jersey Student Learning Standards for English Language Arts Companion Standards	

Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.5.CI.3	Participate in a brainstorming session with individuals with diverse perspectives to expand one's thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience

Instructional Focus

Unit Enduring Understandings

- Matrices are an effective method for solving simple and complex systems of equations.
- Utilize technology (graphing calculator or Desmos) to investigate and extend applications of functions.
- Develop a problem-solving repertoire and be able to choose the appropriate method to solve real world problems.

Unit Essential Questions

- What is the most efficient way to solve a system of equations?
- How can matrices be used to represent complex real world situations?

Content Understandings

- Matrices are rectangular arrays of numbers, organized in rows and columns, used to represent and manipulate data in various fields such as mathematics, computer science, physics, and engineering.
- Basic matrix operations, including addition, subtraction, and scalar multiplication, to combine and manipulate matrices.
- Matrix multiplication, including the requirement for the number of columns in the first matrix to be equal to the number of rows in the second matrix.
- Calculate the determinant of a square matrix and understand its significance in determining invertibility and other properties of matrices.
- The inverse of a square matrix is significant in solving systems of equations.
- Inverse matrices and RREF can be used to solve systems of linear equations.

Content Questions

- What is a matrix?
- How do we identify the dimensions of a given matrix?
- What are some common uses of matrices in various fields?
- How is matrix multiplication defined, and why is it different from scalar multiplication?
- What is the determinant of a matrix, and what does it tell us about the matrix?
- What is an inverse matrix, and how do you find it? Why is it important?
- How can matrices be used to solve systems of linear equations? What is the relationship between matrices and the coefficients of the equations?
- How do row operations help in finding the reduced row echelon form (RREF) of a matrix? Why is RREF useful?

- Can all matrices be inverted? Why or why not?

Objectives

We are learning to/that:

- Add, subtract, and multiply matrices
- Calculate the determinant of a matrix
- Determine the inverse of a matrix
- Solve systems by inverse matrices
- Solve systems by RREF
- Model real world real world situations using matrices
- Identify the most efficient was to solve systems of equations

Evidence of Learning

☑ Formative Assessment

☑ Summative Assessment

☑ Alternative Assessment

☑ Benchmark

Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations

Resources

Core Text: Integrated III, Big Ideas, Larson, Boswell, 2016