West Windsor-Plainsboro Regional School District
Pre-Calculus Honors
June 2023

Math Equity Statement
ALL learners should have access to rigorous, high-level mathematical content in an environment where risk-taking, deep conceptual understanding, and growth mindset are the norm.

Core Beliefs:
Our district's strategic goals prioritize teaching and learning from a productive standpoint. Building upon the principles of *Catalyzing Change in High School Mathematics*, we aim to cultivate equitable mathematics practices and shift from deficit-based to productive beliefs. According to the National Council of Teachers of Mathematics (NCTM, 2020), "Mathematics education must be equitable, ensuring that each and every student has access to high-quality mathematics teaching and learning opportunities." Our objective is for every student to perceive themselves as capable, knowledgeable, and meaning-makers in mathematics.

Drawing from *Catalyzing Change* and *Mathematical Mindsets* by Jo Boaler (2016), we embrace the following beliefs:

- All students are capable of learning mathematics at high levels.
- All students will progress on their mathematical journey.
- Developing a growth mindset is essential for learning.
- Visual and deep thinking enhance mathematical understanding.
- Mathematics learning is fostered through discourse and collaboration.
- Mistakes are integral to the learning process.

Math Workshop:
Catalyzing Change states that teaching should provide opportunities for each and every student to develop a positive mathematical identity, recognizing their own mathematical abilities and potential. The Math Workshop instructional model enables meaningful mathematics engagement, reflection, and the realization of students' potential as mathematicians. By incorporating student choice, problem-solving, targeted small group instruction, and deliberate practice of critical grade-level concepts (Lempp, 2017), Math Workshop creates an environment where students feel comfortable taking intellectual risks. Sienna (2009) outlines four values to support students in taking risks and fostering discourse, which include:

- Valuing the thinking process alongside correct answers.
- Valuing problems that allow for multiple solutions.
- Valuing inquisitive responses.
- Valuing tolerance for mistakes (Sienna, 2009, p. 68).

Math Workshop embraces these values and fosters a supportive, collaborative learning environment for all students. It is the instructional model employed by our dedicated teachers.

Unit 1: Trigonometry	
Content Area: Mathematics	
Course & Grade Level: Pre-Calculus Honors; 10-11	
Summary and Rationale	
Trigonometric functions were developed from ratios within right triangles and are essential in modeling periodic behavior. Solving real world situations in a geometrical context starts by establishing a foundation in the unit circle, which subsequently leads to the development of writing trigonometric functions to model these situations. This unit is designed to expand upon right triangle trigonometry in order to develop a graphical sense of trigonometric functions. After developing a knowledge base for trigonometry, an in-depth algebraic study of trigonometric expressions is examined. A major component of analytic trigonometry is solving equations involving trigonometric expressions. Algebraic relationships, such as inverses and identities, are explored for the purpose of solving these equations. Trigonometric identities are useful in simplifying complex trigonometric expressions into more manageable forms. Trigonometry is also used in many real world contexts. These contexts normally involve oblique (non-right) triangles. Exploration of trigonometric concepts in a wider spectrum of geometry, including surveying and navigation will also be included in this unit.	
Recommended Pacing	
Approximately 65 days	
New Jersey Student Learning Standards for	
Standard: Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: F-IF.B Interpret functions that arise in applications in terms of the context	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship
5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes
Standard: F-IF. C Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude
Standard: F-TF.A Extend the domain of trigonometric functions using the unit circle	
CPI #	**Cumulative Progress Indicator (CPI)**

1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle
2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle
3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π−x, π+x, and 2π–x in terms of their values for x, where x is any real number.
4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions
Standard: F-TF.B Model periodic phenomena with trigonometric functions	
CPI #	**Cumulative Progress Indicator (CPI)**
7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Standard: F-TF.C Prove and apply trigonometric identities	
CPI #	**Cumulative Progress Indicator (CPI)**
8	Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Standard: F-BF.B Build new functions from existing functions	
CPI #	**Cumulative Progress Indicator (CPI)**
4	Find inverse functions
Standard: G-SRT.C Define trigonometric ratios and solve problems involving right triangles	
CPI #	**Cumulative Progress Indicator (CPI)**
8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems
Standard: G-SRT.D Apply trigonometry to general triangles	
CPI #	**Cumulative Progress Indicator (CPI)**
9	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10	Prove the Laws of Sines and Cosines and use them to solve problems.
11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
New Jersey Student Learning Standards for Computer Science and Design Thinking	

CPI #	Cumulative Progress Indicator (CPI)
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience
Interdisciplinary Standards Science	
HS-PS4-1	Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media

Instructional Focus

Unit Enduring Understandings

- Degrees are not always the best unit for measuring an angle.
- Graphs help us describe and interpret real-world phenomena.
- Inverse functions are essential to solving problems.
- Verifying the correctness of solutions through a variety of methods provides a better understanding of the real-world context.
- The calculator and other technologies are tools to supplement and clarify mathematical thinking, and answers from the calculator need to be anticipated and interpreted appropriately.

Unit Essential Questions

- What is the best way to measure an angle?
- How does the study of trigonometry relate to real-world geometry?
- When is an inverse helpful?
- What constitutes a good answer?

Content Understandings

- Degree and radian measurement of an angle are used in different contexts of trigonometry.
- Trigonometric graphs offer a Cartesian representation of the periodic nature generated from unit circle values.
- Cyclic and oscillatory systems can be modeled by sinusoidal functions
- Identities help solve trigonometric equations
- The laws of sines and cosines can be used together to solve problems involving triangles.
- The Cartesian system considers counterclockwise as positive orientation.
- An understanding of right-triangle trigonometry can be used to develop strategies for working with oblique triangles.

Content Questions

- What is the mathematical significance of the unit circle?
- What is the relationship between degree and radian measurement?
- How do we interpret the key components (i.e. amplitude, period, phase shift, vertical shift) of the graph of a trigonometric function?
- What is the importance of the Euclidean and Cartesian approaches to trigonometry?
- How are the six trigonometric functions related?
- Why do we restrict the domains of trigonometric functions in order to determine the inverse functions?
- How can the compositions of trigonometric functions and their inverses be evaluated with precision?
- How are the laws of sines and cosines applied in real-world applications?
- How do you use relationships between trigonometric functions to solve more complex trigonometric equations?
- How can two (or more) trigonometric expressions be verified as equivalent?
- What algebraic techniques are useful in solving trigonometric equations?
- How can we use our analysis of trigonometric functions to solve geometric problems?

Objectives

We are learning to/that:

- Determine arc length and area of a sector of a circle in both degrees in radians.
- Work with angles measured in both degrees and radians and choose the appropriate measure based on the context of the problem.
- Find the value of trigonometric functions of acute angles in a right triangle
- Determine exact values of trigonometric functions of unit circle angles
- Determine the domain, range, zeros, amplitude, phase shift and period of sinusoidal functions
- Graph sinusoidal functions using transformations
- Model real-world phenomena using sinusoidal functions and use models to solve real-world problems
- Understand the connection between the 6 trigonometric functions and their inverses
- Evaluate the exact value of inverse sine, cosine, tangent functions
- Use a calculator to evaluate inverse sine, cosine, tangent functions
- Find the exact value of expressions involving the inverse trig functions, as well as find approximations using the calculator
- Simplify trigonometric expressions and establish/verify identities
- Prove and apply the sum, difference, double-angle, half-angle formulas for sine, cosine, and tangent functions
- Use the sum and difference formulas involving inverse trigonometric functions.
- Properly interpret the calculator output in order to determine the solution(s) within the appropriate domain.
- Solve trigonometric equations with and without the use of a calculator
- Solve and apply problems using right triangle trigonometry
- Prove Law of Sines and Law of Cosines
- Use Law of Sines and Law of Cosines to solve triangles
- Apply Law of Sines and Law of Cosines to real-world problems
- Determine when it is appropriate to use Law of Sines (AAS, ASA, SSA Triangles) and Law of Cosines (SAS, SSS Triangles)
- Derive formulas for the area of a triangle in the SAS
- Find the area of a triangle in the SAS (using the sine function).
- Apply Law of Sines/Cosines to surveying and navigation problems.

Evidence of Learning
☑ Formative Assessment
☑ Summative Assessment
☑ Alternative Assessment
☑ Benchmark
Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations
Resources
Core Text: Precalculus, Enhanced with Graphing Utilities, Sullivan, 2017

<table>
<tr><th colspan="2">Unit 2: Polar Coordinates & Equations</th></tr>
<tr><td colspan="2">Content Area: Mathematics</td></tr>
<tr><td colspan="2">Course & Grade Level: Pre-Calculus Honors; 10-11</td></tr>
<tr><th colspan="2">Summary and Rationale</th></tr>
<tr><td colspan="2">The polar coordinate system begins to build upon the idea of the unit circle. The concept of establishing a point using the distance from the origin (pole) and an angle in standard position is vastly different from establishing a point using horizontal and vertical components. The polar coordinate system is appreciated through its connections with both the rectangular and complex coordinate systems. This allows for the development of conversions to move between each system.</td></tr>
<tr><th colspan="2">Recommended Pacing</th></tr>
<tr><td colspan="2">Approximately 14 days</td></tr>
<tr><th colspan="2">New Jersey Student Learning Standards for</th></tr>
<tr><td colspan="2">Standard: Standards for Mathematical Practice</td></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>1</td><td>Make sense of problems and persevere in solving them.</td></tr>
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<tr><td>8</td><td>Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Standard: N-CN.B Represent complex numbers and their operations on the complex plane.</td></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>4</td><td>Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</td></tr>
<tr><th colspan="2">New Jersey Student Learning Standards for English Language Arts
Companion Standards</th></tr>
<tr><td colspan="2">Standard: Technical Reading</td></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>RST.9-10.7</td><td>Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.</td></tr>
<tr><th colspan="2">New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills</th></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>9.4.12.CT.2</td><td>Explain the potential benefits of collaborating to enhance critical thinking and problem solving.</td></tr>
<tr><td>9.4.12.CI.1</td><td>Demonstrate the ability to reflect, analyze, and use creative skills and ideas.</td></tr>
<tr><td>9.4.12.TL.3</td><td>Analyze the effectiveness of the process and quality of collaborative environments.</td></tr>
<tr><th colspan="2">New Jersey Student Learning Standards for Computer Science and Design Thinking</th></tr>
<tr><td>CPI #</td><td>Cumulative Progress Indicator (CPI)</td></tr>
<tr><td>8.2.12.EC.3</td><td>Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience</td></tr>
</table>

Instructional Focus
Unit Enduring Understandings
- The underlying structure of the Cartesian, Polar, and other coordinate systems of mathematics relates to the interplay between graphical and algebraic representations. As problems become more complex, there are many ways to solve them. Mathematicians look for the most efficient method.   - The underlying structure of the Cartesian, Polar, and other coordinate systems of mathematics relates to the interplay between graphical and algebraic representations.
Unit Essential Questions
- What are the benefits of using non-rectangular coordinate systems?   - How are the rectangular and polar coordinate systems related, algebraically and graphically?   - How does the study of trigonometry and polar graphs relate to real-world phenomena?
Content Understandings   - The polar coordinate system describes a location in space by its distance from a given point called the pole and its angular rotation measured from the polar axis, and can be described using multiple representations.   - Formulas can be used to convert between rectangular and polar equations.   - Polar coordinates make working with some equations much more simple, and are widely used in applications involving navigation.   - All solutions of a system of polar equations are represented by an intersection of the graphs, but intersections may not be revealed as solutions when solved algebraically when an intersection occurs at the pole.
Content Questions   - How is a location in space described in the polar coordinate system?   - How are rectangular equations related to polar equations?   - What is the value in studying the polar coordinate system?   - What is the distinction between the intersections of two polar graphs and solutions to the corresponding system of polar equations?
Objectives
Students will know: - Terms: pole, polar axis, polar coordinates, limaçon, rose curves, cardioid, lemniscate, polar form   **We are learning to/that:**   - Plot points using polar coordinates   - Use polar coordinates to describe multiple representations of a single point in space (positive and negative r, coterminal angles)   - Transform between polar and rectangular coordinates   - Graph and identify polar equations by converting to rectangular equations   - Test polar equations for symmetry   - Graph polar equations (by hand and with a graphing calculator) including rose curves, limaçons, circles, lemniscates, and lines.   - Graph a polar curve over a given domain   - Write the equation of a polar curve given its graph   - Find points of intersection of polar equations & solutions to polar systems
Evidence of Learning
☑ Formative Assessment

☑ Summative Assessment
☑ Alternative Assessment
☑ Benchmark
Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations
Resources
Core Text: Precalculus, Enhanced with Graphing Utilities, Sullivan, 2017

Unit 3: Conic Sections	
Content Area: Mathematics	
Course & Grade Level: Pre-Calculus Honors; 10-11	
Summary and Rationale	
The study of conic sections is a combination of algebra, geometry, and trigonometry. Defined as the intersection of a double napped right circular cone and a plane, each type of conic section can be represented as an equation in the Cartesian or polar plane. These conics (ellipses, parabolas, and hyperbolas) are applied in real-world situations, such as orbits of planets, flashlights, and satellites. Since conics are not always functions, they can be defined using parametric equations.	
Recommended Pacing	
19 days	
New Jersey Student Learning Standards for	
Standard: Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: G-GPE.A Translate between the geometric description and the equation for a conic section	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2	Derive the equation of a parabola given a focus and directrix.
3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
New Jersey Student Learning Standards for Computer Science and Design Thinking	

CPI #	Cumulative Progress Indicator (CPI)
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience

Instructional Focus

Unit Enduring Understandings

- The underlying structure of analytic geometry relates to the interplay between algebra and geometry.
- Mathematicians use many different graphing systems to analyze the relationship between variables, including rectangular, polar and parametric.

Unit Essential Questions

- Which method, geometric or algebraic, is more efficient to solve a problem?
- How do conic sections model real-world phenomena that cannot be modeled by a function?
- How can we use the reflective properties of conic sections to solve real world problems?
- What is the best way to define a curve?

Content Understandings

- A conic section is a curve obtained by intersecting a cone with a plane.
- Curves can be created by defining its components independently, based on a given parameter.
- The geometric definition of conic sections leads to an algebraic equation, defining the conic in the coordinate plane, as a locus of points.
- Each conic section has different characteristics and formulas that help us solve various types of problems.
- Conics may also be described as plane curves that are the paths of a point moving so that the ratio of its distance from a fixed point to the distance from a fixed line is a constant.
- We can use the trigonometric identities to convert the equation for a conic from polar to rectangular form.
- Conic sections can be represented by parametric equations and applied to projectile motion problems.

Content Questions

- How are the four conic sections related?
- What algebraic techniques are useful in analyzing conic sections?
- What do the key components (i.e. respective axes, foci, vertices, center, directrix, eccentricity) of the graph of a conic tell us?
- How can we use our analysis of conic sections to model geometric problems?
- How can we find parametric equations for curves defined by rectangular equations?
- When is it advantageous to define curves parametrically?

Objectives

Students will know:

- Terms: latus chord, directrix, center, focus, eccentricity, definition of a conic, standard form, vertex, major axis, minor axis, transverse axis, conjugate axis , asymptotes
- Equations: General Form of Conic, Standard Form of Circle, Ellipse, Parabola, and Hyperbola.
- Parametric equations of a curve

We are learning to/that:

- Compare and contrast the equations of parabolas, circles, ellipses and hyperbolas.
- Graph the equations of parabolas, circles, ellipses and hyperbolas.
- Solve applications involving parabolas, circles, ellipses and hyperbolas.
- Identify a conic from its general form.
- Determine the type of conic section from the Cartesian equation and subsequently complete the square to rewrite the equation in standard form.
- Determine the equation of specific conics given particular characteristics; i.e. foci, equations of asymptotes,

vertices ● Determine the eccentricity of a conic section and how it affects the shape of the graph. ● Graph parametric equations with and without a graphing calculator ● Find a rectangular equation for a curve defined parametrically ● Use time as a parameter in parametric equations ● Find parametric equations for curves defined by rectangular equations
Evidence of Learning
☑ Formative Assessment
☑ Summative Assessment
☑ Alternative Assessment
☑ Benchmark
Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations
Resources
Core Text: Precalculus, Enhanced with Graphing Utilities, Sullivan, 2017

Unit 4: Functions and Graphs	
Content Area: Mathematics	
Course & Grade Level: Honors Precalculus, grades 10 & 11	
Summary and Rationale	
Representation and analysis of relationships among variable quantities and the solutions of problems involving patterns, functions, and algebraic concepts and processes are essential to success in calculus. Different types of functions are reinforced and extended upon in preparation for higher-level mathematics. Functions provide a foundational learning platform for limits and continuity which leads to understanding of the basics of Calculus.	
Recommended Pacing (Minimum)	
days 15 days	
New Jersey Student Learning Standards for	
Standard: Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: A-APR.B Understand the relationship between zeros and factors of polynomials	
CPI #	**Cumulative Progress Indicator (CPI)**
2	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Standard: F-IF.B Interpret functions that arise in applications in terms of the context	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
Standard: F-IF.C Analyze functions using different representations	
CPI #	**Cumulative Progress Indicator (CPI)**
7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
Standard: F-BF.B Build new functions from existing functions	
CPI #	**Cumulative Progress Indicator (CPI)**
4	Find inverse functions.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience

Instructional Focus

Unit Enduring Understandings

- There are different coordinate systems each with advantages and disadvantages depending on the problem you are solving.
- The various members of the families of functions have similarities among and differences between them.
- The calculator, along with other technology, is a tool to supplement and clarify mathematical thinking; answers from the calculator need to be anticipated and interpreted.
- Investigation and exploration are essential to the development of mathematical ideas. The symbolic language of algebra is used to communicate and generalize the patterns in mathematics.
- Algebraic, graphical, and numerical representations can be used to generalize patterns and relationships.
- Mathematical models can be used to describe and quantify physical relationships; Physical models can be used to clarify mathematical relationships.
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra.
- Mathematicians use functions to model, interpret and explain real-life phenomena.
- Rules of arithmetic and algebra can be used together with (the concept of) equivalence to transform equations and inequalities so solutions can be found to solve problems.

Unit Essential Questions

- How can change be best represented mathematically?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?

- How are patterns of change related to the behavior of functions?
- How do we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- What makes an algebraic algorithm both effective and efficient?

Content Understandings

- The underlying structure of the Cartesian, Polar, and other coordinate systems of mathematics relates to the interplay between graphical and algebraic representations.
- That finding the inverse of a function helps to analyze the effects of domain over range and vice versa.
- That exponential and logarithmic functions are inverses of each other.
- That properties of exponential and logarithmic functions are related.
- That Euler's constant is widely used for real life exponential and logarithmic applications.
- That rational functions can be analyzed algebraically and graphically for - continuity and discontinuity, end behaviors, finding graphical intercepts and identifying domain and range.
- That piecewise functions and absolute value functions can be analyzed algebraically to identify - domain, range, discontinuities.

Content Questions

- What are ways of identifying the algebraic equation of a function, given a graph?
- What are ways of graphing a function, when given an equation?
- What are the methods to identify discontinuities for a given function and determine its domain and range?
- What are various real life applications of functions?
- How are limits, asymptotes, and continuity related?
- How are the properties of exponents and logarithms related?
- Why do we find the inverse of a function?
- What are the advantages and disadvantages of various, equivalent forms of rational expressions?

Objectives

Students will know:

Terms: asymptotes, end behavior, discontinuity, multiplicity, degree, intercepts, limit, logarithm, Euler's number (*e*), base, power, exponent, inverse, domain, range, odd function, even function, maximum, minimum, exponents and logarithms

We are learning to/that:

- Determine domain, range, and asymptotes of exponential, logarithmic and inverse functions.
- Model real world situations with exponential and logarithmic functions and use them to make predictions.
- Solve exponential and logarithmic equations using various methods.
- Evaluate expressions containing exponents and logarithms.
- Graph rational functions by finding zeros, asymptotes, y-intercept and exploring end behavior.
- Write an equation of a given rational function graph.
- Determine domain, range, holes and asymptotes of rational functions.
- Perform long division of polynomials to determine end behavior asymptotes.
- Relate rational function graphs to the idea of a limit.
- Determine the most efficient method in solving polynomial and rational equations and inequalities.
- Interpret an absolute value function as a piecewise function.

Evidence of Learning
☑ Formative Assessment
☑ Summative Assessment
☑ Alternative Assessment

☑ Benchmark
Assessment plan includes teacher-designed formative and summative assessments, a district common assessment, self-assessments, and performance tasks. During each common, formative, and summative assessment, teachers will provide alternative assessment opportunities that adhere to 504 and IEP requirements. Alternative assessments are individualized for the needs of all students. Accommodations
Resources
Core Text: Precalculus, Enhanced with Graphing Utilities, Sullivan, 2017

Unit 5: Limits & Continuity	
Content Area: Mathematics	
Course & Grade Level: Pre-Calculus Honors; 10-11	
Summary and Rationale	
The concept of limits is essential for developing the underlying theorems used throughout calculus. The mastery of determining one-sided and two-sided limits analytically, graphically, and numerically sets a foundation for differential and integral calculus. These techniques help evaluate limits, especially those in indeterminate form. The exploration of continuity provides a deeper understanding of how functions work, in addition to special limits, such as infinite limits and limits at infinity.	
Recommended Pacing	
12 days	
New Jersey Student Learning Standards for	
Standard: Standards for Mathematical Practice	
CPI #	**Cumulative Progress Indicator (CPI)**
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Standard: F-IF.B. Interpret functions that arise in applications in terms of the context	
CPI #	**Cumulative Progress Indicator (CPI)**
4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
New Jersey Student Learning Standards for English Language Arts Companion Standards	
Standard: Technical Reading	
CPI #	**Cumulative Progress Indicator (CPI)**
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills	
CPI #	**Cumulative Progress Indicator (CPI)**
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
New Jersey Student Learning Standards for Computer Science and Design Thinking	
CPI #	**Cumulative Progress Indicator (CPI)**
8.2.12.EC.3	Synthesize data, analyze trends, and draw conclusions regarding the effect of a technology on the individual, culture, society, and environment and share this information with the appropriate audience

Instructional Focus

Unit Enduring Understandings

- There are different coordinate systems each with advantages and disadvantages depending on the problem you are solving.
- The various members of the families of functions have similarities among and differences between them.
- The calculator, along with other technology, is a tool to supplement and clarify mathematical thinking; answers from the calculator need to be anticipated and interpreted.
- Investigation and exploration are essential to the development of mathematical ideas. The symbolic language of algebra is used to communicate and generalize the patterns in mathematics.
- Algebraic, graphical, and numerical representations can be used to generalize patterns and relationships.
- Mathematical models can be used to describe and quantify physical relationships; Physical models can be used to clarify mathematical relationships.
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra.
- Mathematicians use functions to model, interpret and explain real-life phenomena.
- Rules of arithmetic and algebra can be used together with (the concept of) equivalence to transform equations and inequalities so solutions can be found to solve problems.

Unit Essential Questions

- What is a limit?

Content Understandings

- The idea of the limit of a function is what connects algebra and geometry to the mathematics of calculus.
- Limits can be determined numerically, graphically, and algebraically.
- Limits can be used to analyze a graph and determine continuity.
- Instantaneous rate of change (velocity) can be determined using limits and average rates of change.

Content Questions

- What are the ways to determine the limit of a function?
- What algebraic techniques are useful in finding limits of a function?
- How can limits be used to analyze a graph and determine continuity of a function?
- How can we find the average rate of change and the instantaneous rate of change for a function?
- How are limits used to describe the end behavior of functions?
- What are various real life applications of limits?

Objectives

Students will know:

- Terms: Limit, One-Sided Limits, vertical and horizontal asymptotes, local linearity, continuous, continuous on an interval, removable, jump, infinite discontinuity,
- Theorems: Limit Laws

We are learning to/that:

- Find the limit graphically, and by analyzing a table of values
- Find the limit of a sum, difference, product and quotient
- Apply the limit laws to evaluate them algebraically
- Find one-sided limits

Evidence of Learning

☑ Formative Assessment

☑ Summative Assessment
☑ Alternative Assessment
☑ Benchmark
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